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MATHEMATICAL ANALYSIS OF HEAT FLOW PROCESSES IN RADIATORS

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Abstract

The present study investigates the mathematical modeling and simulation of heat transfer processes in radiators, which constitute essential elements of modern heating systems. The research addresses both conductive and convective mechanisms of heat transfer, employing Fourier's law to describe heat conduction and Newton's law of cooling to characterize convective heat exchange. Various radiator types including cast-iron, steel, aluminum, and bimetallic are analyzed with respect to their thermal performance, structural characteristics, and operational efficiency. The influence of different connection schemes on fluid flow patterns, temperature distribution, and heat dissipation is examined through COMSOL Multiphysics simulations. The results demonstrate the significant effect of design parameters, such as pipe diameter, number of fins and inlet or outlet arrangements, on thermal performance and energy efficiency. Furthermore, the study discusses technical strategies to enhance heat transfer effectiveness, including hydraulic balancing, thermostatic regulation, and integration with low-temperature heating systems. The findings provide a scientific basis for optimizing radiator design, minimizing energy losses, and improving the overall efficiency of residential and industrial heating systems.

Keywords: Heat transfer, Fourier's law, Newton's law of cooling, convection, Nusselt number.

1. Introduction

A heat supply system is an engineering network designed to generate, distribute, and deliver the required thermal energy to buildings and structures. Typically, a heat supply system consists of a heat source, distribution networks, and consumers.

Knudsen, Fiorentini, and Petersen proposed an EMPC (Economic Model Predictive Control) strategy for managing hydraulic radiators based on estimating heat output without the use of special sensors. Simulation results show that the proxy-based approach achieves almost the same efficiency as traditional measurement-based methods (with approximately 16% cost reduction), while maintaining indoor comfort conditions. This method reduces the need for additional measuring devices, making the system more practical and cost-effective [1]. Molla, Hossain, and Islam modeled the heat exchange process in radiators using nanofluids (CuO, ZnO, Al_2O_3) through the CLBM (Central Lattice Boltzmann Method). According to the study results, nanofluids enhance heat transfer, though efficiency is lower in non-Newtonian fluids. This approach provides an important scientific basis for the efficient design of radiator systems [2]. In the study by Shen, Sun, Wang, Duan, Qin, Li, and Zhou, the performance efficiency of foam metal radiators was experimentally analyzed, and parameter optimization was investigated. The influence of heat exchange surface area, flow velocity, and structural characteristics was compared. The results showed that proper selection of parameters significantly improves the thermal efficiency of the radiator. This approach is practically important for developing energy-saving and high-efficiency radiator systems [3]. Caglar's research examined the effect of using water-based nanofluids on the efficiency of panel radiators. The results showed that the use of nanofluids improves the heat transfer capability of radiators, reduces energy consumption, and enhances the overall performance of the system. This approach provides a significant scientific foundation for ensuring energy efficiency and stable operation in radiator heating systems [4]. In the study conducted by Alescia, Fiorentini, Zanetti, and Scoccia, the role of building thermal mass in achieving energy flexibility when integrated with radiator and floor heating

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systems was investigated. The analysis demonstrated that a building's thermal mass allows for temporal control of energy consumption and enhances demand-side flexibility in the energy system. When comparing radiator and floor heating systems, floor heating provides greater energy flexibility due to thermal inertia, while radiators are characterized by a faster thermal response [5]. Joshi and co-authors studied heat transfer in radiators using graphene-based nano-oxidizing fluids. The research results indicate a significant improvement in radiator performance. This approach is important for energy saving and the enhancement of heating systems [6]. In the research conducted by Jian and colleagues, the problem of improving the efficiency of low-temperature radiator systems was addressed. Using machine learning techniques, they optimized the radiator surface model, achieving more accurate calculations of heat distribution and reduced energy consumption [7]. Giroux and collaborators investigated the effective conversion of heat into radiation through the specialized design of radiators. They demonstrated that by altering the shape and material (based on InAs), energy efficiency can be improved in such systems [8]. Rakoch and his team studied the effect of dissolved oxygen content on the corrosion process in heating radiators made from AK12M2 alloy. Experimental results revealed that an increase in oxygen concentration accelerates alloy degradation, intensifying the corrosion process. These findings are important for selecting durable materials and extending the service life of radiators [9]. Ding, Nord, and Timoudas applied a hybrid approach to predict heat load in low-temperature district heating systems. The study combined physical models with data-driven methods and analyzed cases involving nursing homes. Parameters such as outdoor temperature, internal load profile, and building structural features were compared. Results showed that the hybrid approach improves heat load prediction accuracy, reduces energy consumption, and enhances system efficiency [10]. Moghadam, Bruton, O'Sullivan, and Norton examined the problem of maintaining indoor air quality and thermal comfort in buildings through energy-efficient methods. Their study employed a mechanical ventilation heat recovery system and solar-assisted preheating of incoming air. Various parameters related to air quality, indoor temperature, and energy consumption were compared. The results indicated that such an integrated system reduces heat losses, increases energy efficiency, and creates a comfortable indoor microclimate for occupants [11]. Rahmati and Gheibi showed that design modifications in hot-water radiators significantly improve heat transfer based on experimental and numerical analyses. Gergely et al. emphasized the importance of life-cycle carbon assessment for sustainable HVAC systems. Caglar's study demonstrated that nanofluids enhance radiator heat transfer and reduce energy consumption [12]. Lomas et al. reviewed the effectiveness of domestic heating controls in reducing energy use. Their findings show that well-configured control systems can significantly lower energy consumption, although their performance depends strongly on user behavior. The study highlights the importance of optimizing control strategies to achieve higher energy efficiency in residential heating [13]. Ibastea et al. propose a smart retrofit approach for monitoring and controlling energy consumption in buildings using Web of Things (WoT) technology. Their findings show that WoT-based systems enhance real-time monitoring and improve energy optimization. This approach offers innovative and practical solutions for increasing the energy efficiency of existing buildings [14]. Chunhua Sun, Yanan Liu, Shanshan Cao, Xiaoyu Gao, Guoqiang Xia, Chengying Qi, and Xiangdong Wu propose an integrated control strategy for district heating systems based on load forecasting and indoor temperature measurements. Their findings show that this approach improves the accuracy of heat distribution and significantly reduces energy consumption. The authors emphasize that intelligent control systems play a key role in enhancing the efficiency of district heating networks [15].

Previous studies have investigated radiator and heating system performance from various perspectives, including heat transfer enhancement through design optimization and nanofluid application, as well as energy efficiency improvement using advanced control and load forecasting strategies. These works provide valuable insights into improving thermal performance and reducing energy consumption in radiator-based heating systems.

However, existing studies typically address these aspects independently and offer limited discussion on their combined effects under realistic operating conditions. In this context, the present study builds upon earlier research by providing an integrated analysis of radiator heat transfer behavior and energy performance under practical operating conditions. The study combines thermal performance evaluation with energy efficiency considerations to assess the overall effectiveness of radiator-based heating systems. The obtained results contribute to a better understanding of how radiator characteristics influence energy consumption and support the development of more efficient and practically applicable heating solutions.

Centralized heat supply is considered the most efficient solution for providing thermal energy to multi-story buildings, industrial enterprises, and large

facilities. In such systems, heat is generated by high-capacity equipment and delivered to consumers through main pipelines.

1) Hot Water Boilers – These systems produce hot water with temperatures ranging from 90°C to 150°C, most commonly using natural gas or coal as fuel. Their main advantages are stable operation and the ability to cover wide areas.

2) Thermal Power Plants (TPPs) – These are cogeneration plants that simultaneously produce both electricity and heat. They are highly efficient, as they generate electrical energy in addition to thermal energy.

3) District Heating Centers (DHCs) – Medium-capacity facilities that supply heat to local areas. Their main advantage is that they do not require extensive transportation networks.

Local heat supply is mainly used in individual residential houses, small buildings, or remote areas. These sources generate heat for a single consumer.

1) Small boiler houses – Typically serve a single building or small residential blocks. Hot water is used as the heat carrier.

2) Gas boilers – Modern, compact, and efficient units that use natural gas as fuel. They are equipped with automatic control systems and offer a high level of safety.

3) Solar collectors – A renewable energy source that heats water using solar radiation. They are environmentally friendly and designed for long-term use, though their performance depends on solar intensity.

4) Heat pumps – Generate energy by extracting heat from the air, groundwater, or soil. They are among the most energy-efficient and environmentally sustainable heat sources.

The advantages and disadvantages of heat sources are presented in table 1 below.

Table1

Heat source	Advantages	Disadvantages
Centralized heat supply (TPPs, DHCs, Boiler houses)	-Provides heat to a large number of consumers -Allows centralized management of fuel resources -Ensures stable and continuous heat supply -High efficiency of large-capacity production	-Significant heat losses through main pipelines -High construction and maintenance costs -Low flexibility (difficult to control individual rooms) -Connection to the system is generally limited to urban areas
Local heat sources (small boiler houses, gas boilers, solar collectors, heat pumps)	-Operates autonomously, independent of the central network -Minimal heat losses in the heat carrier -Allows independent control for each building or room -Can operate in an environmentally friendly manner using renewable energy sources	-Requires separate maintenance and operational costs for each facility -Difficult to cover large buildings or complexes -High safety requirements for gas boilers -Performance of solar collectors and heat pumps depends on climatic conditions

A radiator is one of the main components of a heat supply system, and its function is to efficiently transfer heat from hot water or steam into a room. The operating principle of radiators is primarily based on convection and radiation phenomena. Through convection, warm air rises from the bottom upward, allowing the room air to heat up more quickly, while radiation transfers heat directly to the surrounding environment. Radiators are classified into several types, each with its own specific advantages and disadvantages.

Cast-Iron radiators. Cast-iron radiators are the most common and traditional type of heating devices. They are distinguished by their long service life (up to 50 years) and high resistance to rust and corrosion. Due to their high thermal inertia, they heat up slowly but also cool down slowly, helping to maintain a stable indoor temperature. However, their drawbacks include heavy weight and a simple external design, which may not always fit modern interiors (fig1).

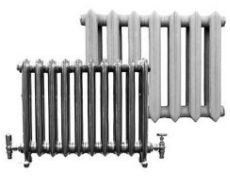


Figure1. Appearance of a cast-iron radiator



Figure 2. Appearance of a steel radiator



Figure 3. Appearance of an aluminum radiator

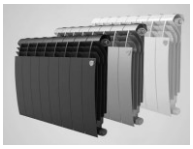


Figure 4. Appearance of a bimetallic radiator

Steel radiators. Steel radiators are characterized by their light weight, quick heating, and compact design. They are also more affordable compared to other types. However, they are prone to corrosion, with an average service life of about 15–20 years. Modern buildings commonly use steel panel radiators (fig 2).

Aluminum radiators. Aluminum radiators have a very high heat transfer coefficient. They are lightweight and produced in modern, aesthetically pleasing designs. However, their performance depends greatly on the quality of the heat carrier: if the water has an aggressive chemical composition, they can corrode quickly. Therefore, they are best suited for clean and well-maintained hot water systems (fig 3).

Bimetallic radiators. Bimetallic radiators consist of steel pipes coated with aluminum, making them highly resistant to pressure and chemical environments. Although they are relatively more expensive, they are durable, efficient, and feature modern designs, making them highly popular among consumers (fig 4).

When selecting a radiator, special attention should be paid to its technical specifications. The heat output (measured in watts or kcal/hour) indicates the amount of heat the radiator can deliver. The working pressure (measured in bar or MPa) shows the radiator's ability to withstand the water or steam pressure within the system. The temperature of the heat carrier (°C) determines the temperature range in which the radiator operates efficiently. The service life reflects the expected duration of use. Proper placement of radiators also has a direct impact on heating efficiency.

2. Physical formulation of the problem

According to CNR 2.04.05-22 – "Heating, Ventilation and Air Conditioning (HVAC)" the following requirements are established. Radiators should primarily be installed under windows, as this helps reduce heat loss and block incoming cold air. If there are no windows, they should be installed near the exterior wall of the room [16]. To ensure proper air circulation (convection) within the room, radiators must not be obstructed by furniture or curtains. In addition, to maintain optimal efficiency, the following installation distances must be observed:

Distance from the floor – 8-12 sm;

Distance from the wall – 3-5 sm;

Distance to the shelf or window sill – 10-15 sm.

In contemporary heat supply systems, the issue of energy efficiency optimization represents a critical and pressing challenge. To enhance the operational performance and thermal efficiency of radiators, the following technical and engineering measures are implemented:

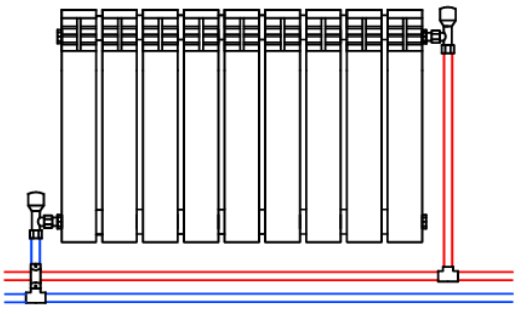
1) Installing thermostatic regulators on radiators to enable automatic heat control;

2) Ensuring hydraulic balancing within the system to guarantee an even distribution of heat to all radiators;

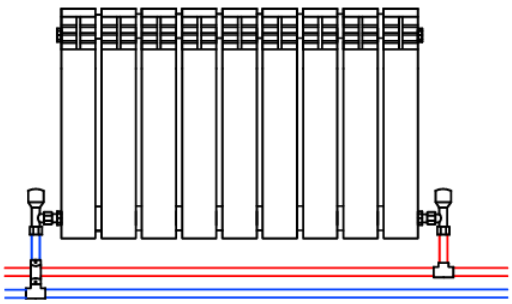
3) Combining low-temperature radiators with underfloor heating systems to improve overall energy efficiency;

4) Applying energy-efficient thermal insulation in building structures and using modern construction materials to minimize heat losses.

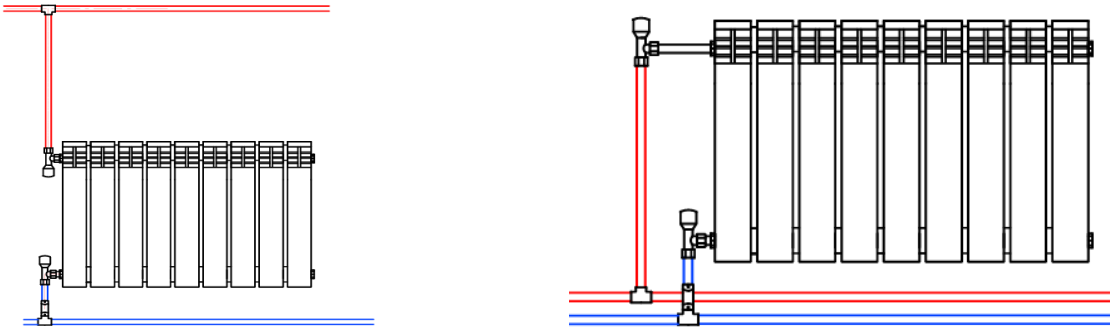
Radiators represent one of the most essential components of a heat supply system, as they play a key role in maintaining a comfortable indoor microclimate. When selecting the type of radiator, it is important to consider technical capabilities, economic efficiency and aesthetic requirements. The various connection types of radiators are illustrated in Figure 5 below.



a) Hot water enters from the top and cold water returns from the bottom



b) Hot water enters from the bottom and cold water returns from the bottom



c) Both the inlet and outlet are located on the same side: hot water enters from the top, and cold water returns from the bottom

Figure 5. Two-pipe heat supply scheme of a radiator

Although bimetallic and aluminum radiators are more commonly used in modern construction, the classic cast-iron radiators are still valued for their stability and durability. At the same time, the problem of predicting heat loads in low-temperature heat supply systems has been actively studied. The connection configuration of hot-water radiators affects the movement of fluid inside the radiator, and it is essential that the fluid circulates through all radiator sections. This ensures full utilization of the heat and provides the room with the required and calculated thermal energy. Accordingly, when setting the parameters for experimental calculations of radiator connections using the COMSOL software, the following conditions were defined the diameter of the radiator pipe is $d = 8\,mm$, and the number of fins (sections) is 14. The hot water inlet configurations are modeled in four different types.

3. Computational domain, mesh and numerical method

The computational domain represents a single radiator element consisting of a horizontal header pipe with a diameter of $d = 8$ mm and 14 vertical fins (sections). A three-dimensional model was generated in COMSOL Multiphysics, assuming incompressible single-phase flow of water and conjugate heat transfer between the fluid and the solid walls. The geometry corresponds to the four connection schemes illustrated in Fig. 6.

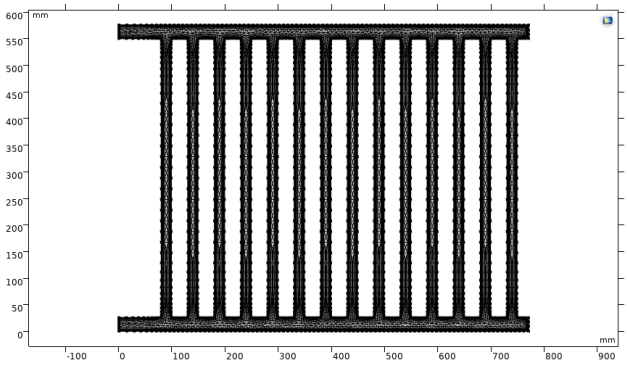


Fig. 6. Computational mesh of the radiator model

The domain was discretized using an unstructured tetrahedral mesh with local refinement near the solid walls and at the inlet and outlet regions in order to accurately resolve the velocity and temperature gradients. Additional boundary-layer elements were employed along the inner pipe walls to capture the near-wall effects. A mesh independence study was carried out by successively refining the grid until the variation in the predicted outlet temperature and total heat flux was below $1-2\%$. The final mesh contained approximately $N \approx$..elements (to be specified), which provided a good compromise between accuracy and computational cost.

At the inlet, a fully developed velocity profile with a prescribed volumetric flow rate (or average velocity) and a constant temperature $T_{\text{in}} = 70–75°\text{C}$ was imposed. At the outlet, a zero-gauge pressure (p = 0) boundary condition was applied, allowing the flow to adjust freely. No-slip conditions (u = 0) were used at all solid walls. Heat transfer through the radiator walls to the ambient air was modeled by a convective boundary condition of the form

$$-k\nabla T \cdot \mathbf{n} = h\,(T_{\text{wall}} - T_{\infty}), \qquad (1)$$

where $h$is the overall heat transfer coefficient between the radiator surface and the room air, and $T_\infty$is the indoor air temperature.

The initial condition for all transient simulations assumed a uniform temperature field equal to the ambient room temperature and zero velocity in the fluid domain. In the present work, stationary solutions were obtained by integrating the governing equations until steady-state convergence was reached.

The governing equations were discretized using the finite element method. A segregated solution strategy was adopted, where the velocity, pressure, and temperature fields were solved iteratively until the relative residuals were below 10^{-5}. The time integration (for transient runs, if any) was performed using an implicit scheme with automatic time-step control. The simulations were carried out for all four inlet–outlet configurations under identical operating conditions to enable a consistent comparison of the resulting temperature distributions and heat fluxes.

4. Mathematical formulation of the problem

Based on Fourier's law, the mathematical formulation of the heat transfer process can be expressed as the heat flux through any surface. In constructing the mathematical model, the calculation is determined using the following equations.

For an incompressible Newtonian fluid, the conservation of mass is expressed by the continuity equation

$$\nabla \cdot \mathbf{u} = 0, \tag{2}$$

where $\mathbf{u} = (u, v, w)$is the velocity vector.

The conservation of momentum in the fluid domain is governed by the incompressible Navier–Stokes equations

$$\rho(\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla p + \mu\nabla^2\mathbf{u} + \rho\mathbf{g}, \tag{3}$$

where $\rho$is the fluid density, $p$is the pressure, $\mu$is the dynamic viscosity, and $\mathbf{g}$is the gravitational acceleration vector. The first term on the left-hand side represents the convective transport of momentum, whereas the terms on the right-hand side account for pressure forces, viscous diffusion and body forces, respectively.

In the present study, the flow regime inside the radiator was turbulent. Therefore, the Reynolds-averaged Navier–Stokes (RANS) equations were solved, in which the instantaneous velocity is decomposed into mean and fluctuating components, $\mathbf{u} = \bar{\mathbf{u}} + \mathbf{u}'$. The effect of turbulence was modeled using the standard k–ε model, where the turbulent viscosity $\mu_t$is computed from the turbulent kinetic energy $k$and its dissipation rate ε. In this case, an effective viscosity $\mu_{\text{eff}} = \mu + \mu_t$appears in the momentum equations.

The temperature field in the fluid is obtained from the energy equation

$$\rho c_p(\mathbf{u} \cdot \nabla T) = \nabla \cdot (k\nabla T) + Q, \tag{4}$$

The coupled system consisting of the continuity, momentum and energy equations, together with the appropriate boundary and initial conditions, fully describes the convective–conductive heat transfer in the radiator.

Heat balance equation:

$$\rho C_p u \cdot \nabla T + \nabla \cdot q = d_Z Q + q_0 + d_Z Q_p + d_Z Q_{vd}. \tag{5}$$

The given equation describes the heat balance within a specified volume (or layer). From this, it follows that the left-hand side of the equation represents the heat diffusion (or heat transfer) process within the medium..

Here, $d_Z \rho C_p u \cdot \nabla T$ – It represents convective heat transfer, which is the movement of heat due to fluid flow. This term expresses the change in heat energy per unit volume.

d_Z – the thickness of the studied volume $[m]$,

ρ – density $[kg/m^3]$,

C_p – specific heat capacity at constant pressure $[J/kg \cdot K]$,

u – velocity vector of the flow$[m/s]$,

∇T – temperature gradient $[K]$,

$\nabla \cdot q$ – thermal conduction, representing the transfer of heat from one location to another due to a temperature difference.

The right-hand side of the equation represents the heat energy entering the given volume from external sources. This term corresponds to the divergence of the heat flux.

Here, $d_Z Q$ – volumetric density of heat sources,

q_0 – heat flux entering from the upper and lower boundaries,

$d_Z Q_p$ – heat generated due to internal pressure (P),

$d_Z Q_{vd}$ – heat generated due to viscous forces (dissipation).

Fourier's law describes the conduction of heat within a medium. It states that the heat flux density q is proportional to the negative temperature gradient:

$$q = -d_z k \nabla T \tag{6}$$

This equation represents one of the fundamental laws of heat transfer and allows the

determination of the heat flux q in the first equation presented above.

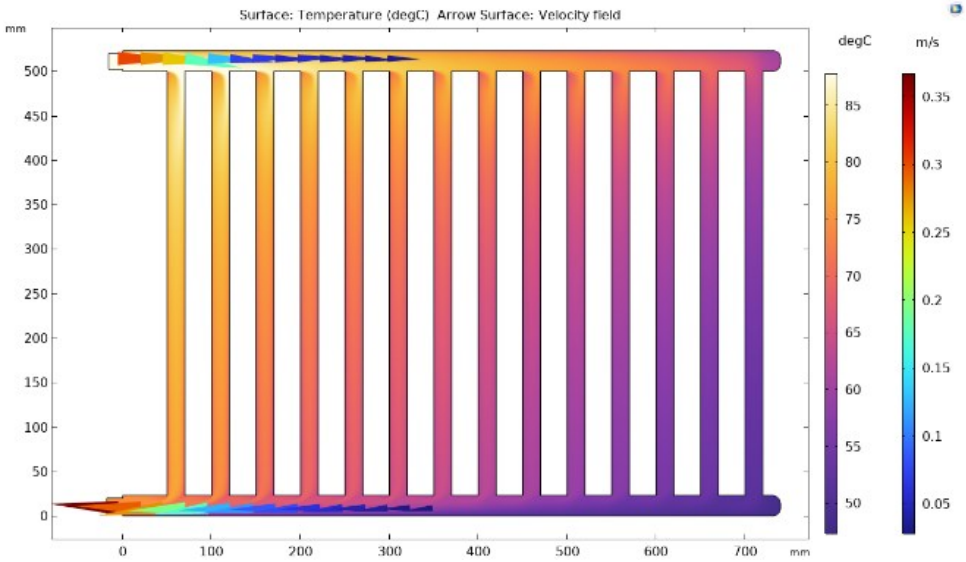
Here, q – heat flux vector, indicating the direction and rate of heat transfer from one location to another $[W/m^2]$,

k – thermal conductivity coefficient, a material property that measures the ability to conduct heat $[W/m \cdot K]$,

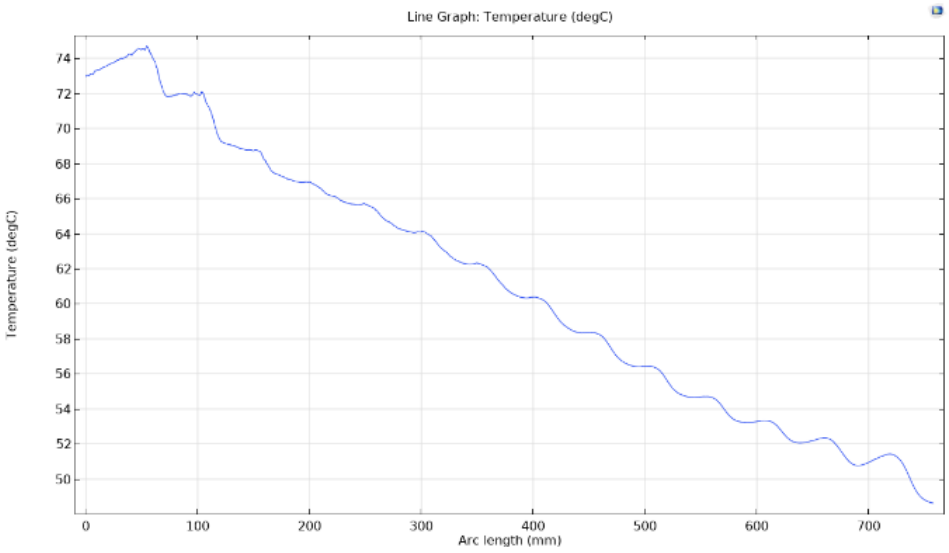
The negative sign $(-)$ indicates that heat always flows from regions of higher temperature to lower temperature.

5. Results

In this figure, the results of computer simulation of heat exchange and fluid flow are presented for a one-sided flow scheme,



a) Simulation of heat exchange and fluid flow



b) Temperature decreases along the fluid flow direction

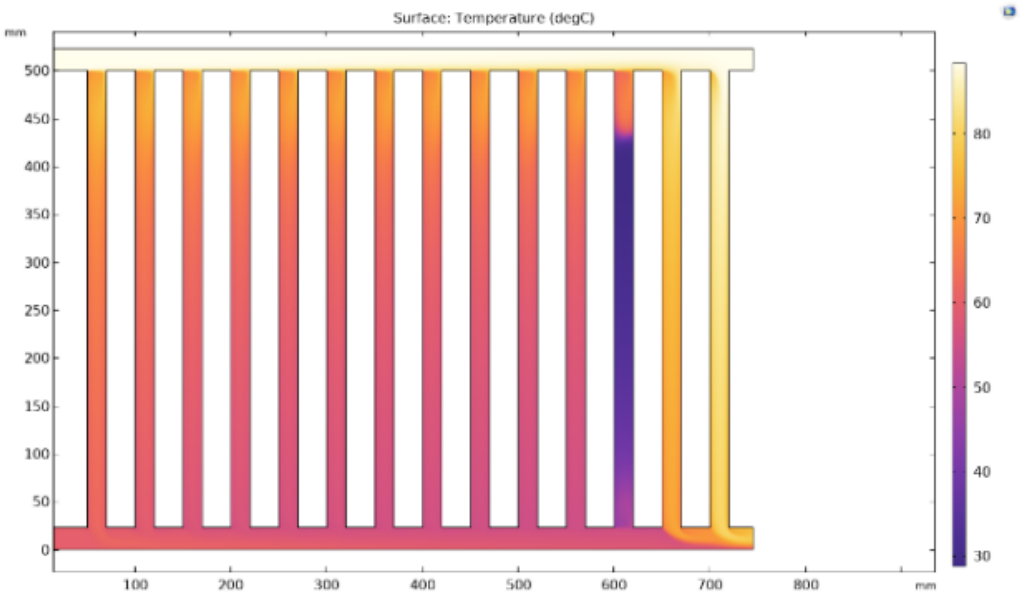
Figure 7. Hot water enters from the top, and cold water returns from the bottom (one-sided connection)

where hot water enters from the top and cooled water exits from the bottom.

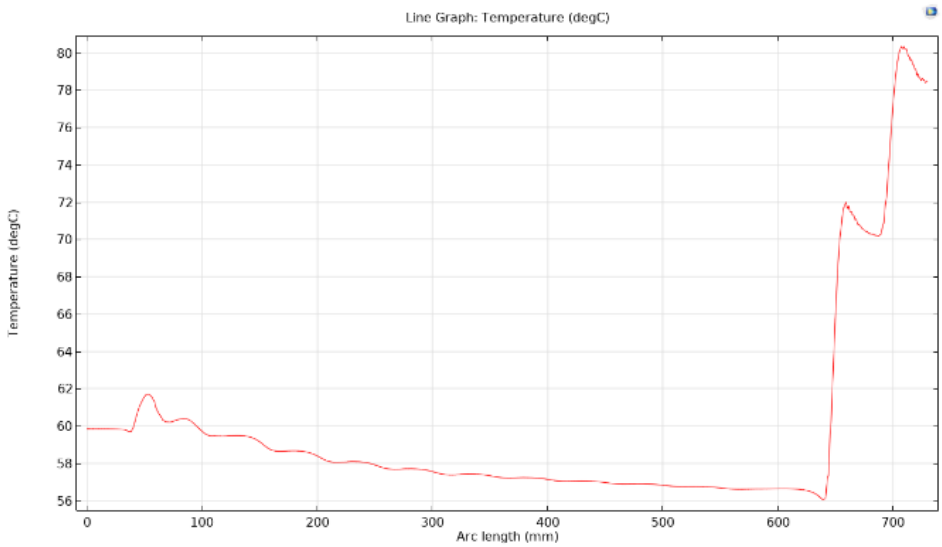
In Figure (a), the heat exchange process within the pipe system is illustrated, showing the temperature distribution and flow velocity field using colored gradients. The results indicate that although the temperature near the hot water inlet remains high $(70 - 75\ ^\circ C)$, the fluid temperature gradually decreases along the pipe due to heat exchange with the surrounding environment. At the same time, local variations in flow velocity are observed, which are explained by hydraulic resistance and the redistribution of flow across the pipe cross-section.

In Figure (b), the temperature change along the fluid flow direction is shown. Analysis of the graph demonstrates that the hot water starts at

approximately $74\,°C$ at the inlet and decreases to around $50\,°C$ at the pipe outlet. This gradual temperature drop confirms the continuous nature of the heat exchange process and highlights the significant temperature difference between the inlet and outlet.



Simulation of heat exchange and fluid flow



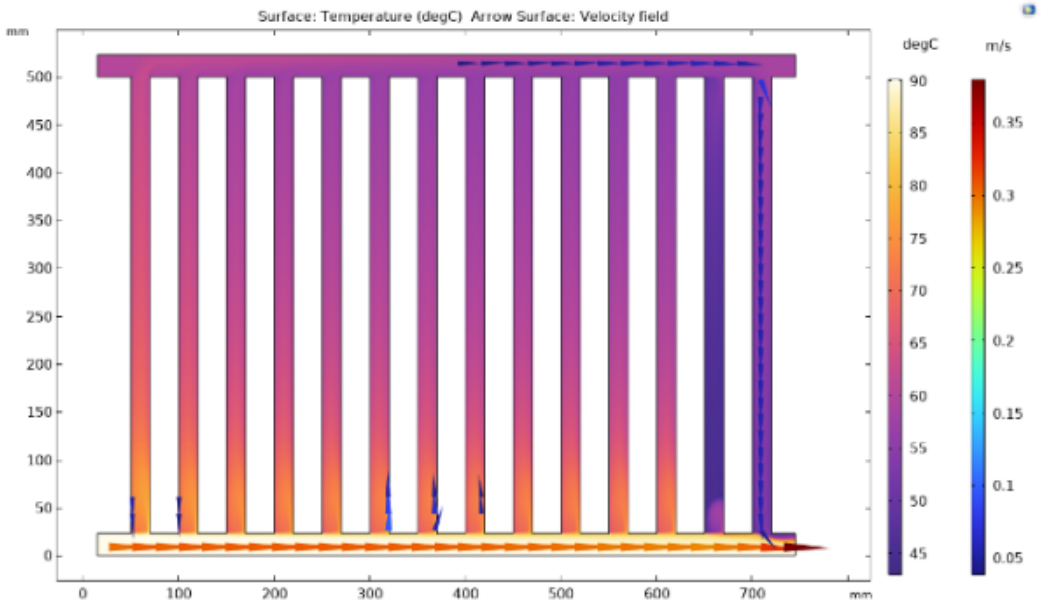
Temperature decreases along the fluid flow direction

Figure 8. Hot water enters from the top and cold water returns from the bottom (hot water inlet on the left, outlet on the right)

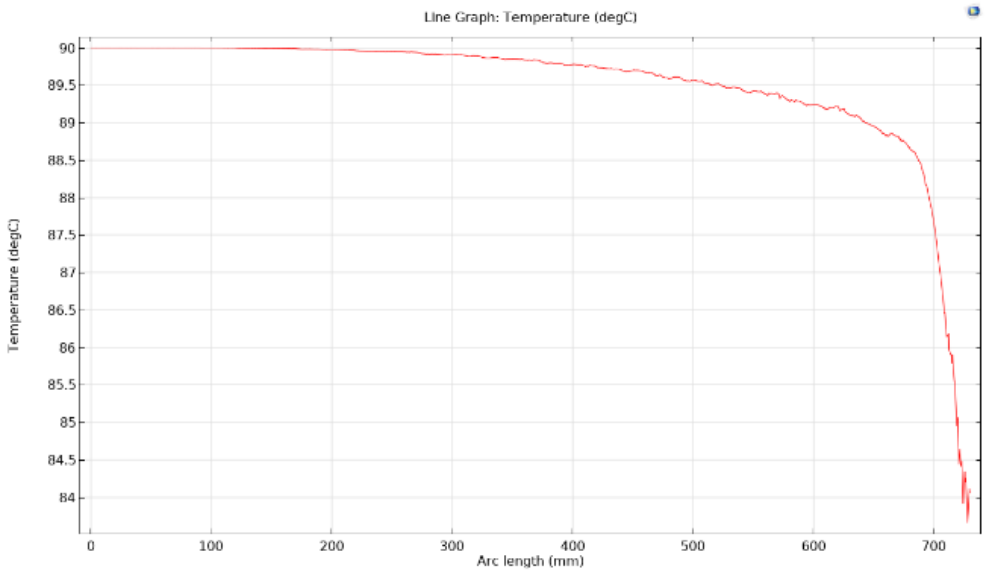
a) Simulation of heat exchange and fluid flow. This graph illustrates the temperature distribution across the surface and the vector field of fluid flow. Colors represent temperature: red and yellow indicate high temperatures (up to $500\,°C$), while blue and purple indicate low temperatures (from $50\,°C$). The arrows show the direction and velocity of the fluid ($0-0.25\,m/s$). The scheme depicts hot water entering from the left and cold water exiting from the right, enhancing heat exchange efficiency. The simulation was conducted in COMSOL, taking into account both convection and conduction, and models the surface heat losses.

b) Temperature decreases along the fluid flow direction. The line graph shows how temperature changes along the arc length (distance, mm). The initial temperature starts at $72\,°C$ and decreases to $50\,°C$ over $700\,mm$, with a roughly linear decline and minor fluctuations. This represents heat loss along the fluid flow, for example, the cooling of water inside the radiator. The graph is a cross-sectional result of the simulation, dependent on thermal conductivity and flow velocity. These results are used to evaluate

hydraulic and thermal balance and can improve radiator efficiency by $20-30$ %.



Simulation of heat exchange and fluid flow

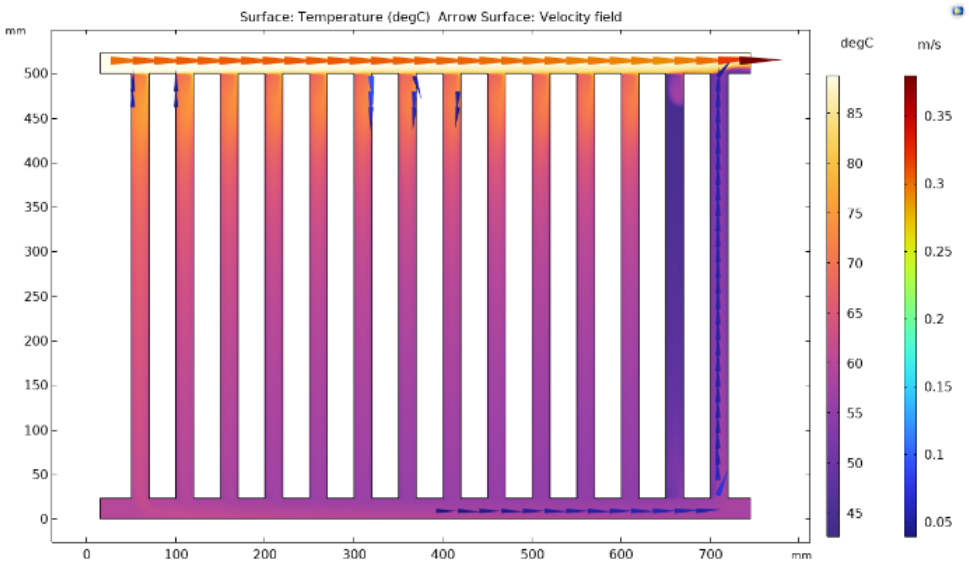


Temperature decreases along the fluid flow direction

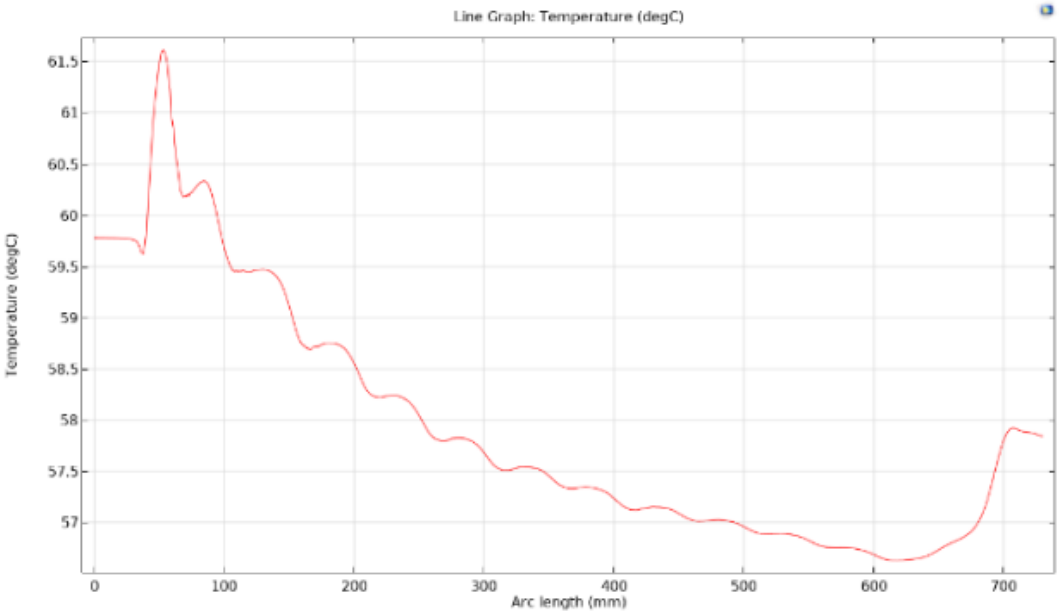
Figure 9. Hot water enters from the bottom, and cold water returns from the bottom (bottom inlet and bottom outlet scheme)

a) Simulation of heat exchange and fluid flow. This graph shows the temperature distribution across the surface and the vector field of fluid flow. Colors indicate temperature: red and dark pink represent high temperatures (up to $500\,°C$), while light pink and blue represent low temperatures (from $50\,°C$). The arrows show the direction and velocity of the fluid ($0-0.25\,m/s$). The scheme depicts hot water entering from the bottom and cold water exiting from the top, enhancing convection and conduction effects inside the radiator. The simulation was performed in COMSOL, modeling heat transfer and turbulent flow to represent the surface temperature distribution.

b) Temperature decreases along the fluid flow direction. The line graph shows temperature variation along the arc length (distance, mm). The initial temperature starts at $89{,}5\,°C$ and decreases to $84\,°C$ over $700\,mm$, with a rapid drop followed by stabilization. This reflects heat exchange along the fluid flow, such as the cooling of water inside the radiator. The graph is a cross-sectional result of the simulation, dependent on flow velocity and thermal conductivity. These results are used to evaluate hydraulic and thermal efficiency and can reduce heat losses by $5-10$ %.



a) Simulation of heat exchange and fluid flow



b) Temperature decreases along the fluid flow direction

Figure 10. Hot water enters from the top, and cold water returns from the top (top inlet and top outlet scheme)

a) Simulation of heat exchange and fluid flow. This graph illustrates the temperature distribution across the surface and the vector field of fluid flow. Colors indicate temperature: red and yellow represent high temperatures (up to $500\,°C$), while purple and blue represent low temperatures (from $50\,°C$). The arrows show the direction and velocity of the fluid ($0-0.35\,m/s$). The scheme depicts hot water entering from the top and cold water exiting from the top, promoting turbulent flow inside the radiator and enhancing heat transfer efficiency. The simulation was conducted in COMSOL, taking into account both convection and conduction to model the radiator's heat exchange.

b) Temperature decreases along the fluid flow direction. The line graph shows the temperature variation along the arc length (distance, mm). The initial temperature starts at $60{,}5\,°C$ and decreases to $57\,°C$ over $700\,mm$, with a roughly linear decline and minor fluctuations. This represents heat loss along the fluid flow, such as the cooling of water inside the radiator. The graph is a cross-sectional result of the simulation, dependent on thermal conductivity and flow velocity. These results are used to evaluate hydraulic and thermal balance and can improve radiator efficiency by $10-20\,\%$.

6. Conclusions

The present study provides a comprehensive analysis of heat transfer processes in radiators, combining theoretical, numerical, and practical perspectives. The mathematical formulation based on Fourier's law of heat conduction and Newton's law of cooling enabled precise modeling of both conductive and convective heat transfer within radiator systems. The simulations conducted in COMSOL demonstrated how different radiator connection schemes affect fluid flow, temperature distribution, and heat transfer efficiency. Results indicate that proper design of radiator connections, consideration of hydraulic and thermal balancing, and selection of suitable materials significantly influence overall energy efficiency. For example, one-sided, top-bottom, bottom-bottom, and top-top inlet/outlet schemes exhibit distinct temperature gradients and flow patterns, impacting heat dissipation and operational performance. The study confirms that mathematical modeling and computer simulations are powerful tools for optimizing radiator performance, minimizing energy losses, and predicting thermal behavior under various operating conditions. The findings also highlight the importance of parameters such as fluid velocity, pipe diameter, number of radiator fins, and the thermal properties of materials in achieving effective heat transfer.

Future research plans include computer modeling of heat supply systems, broad implementation of numerical simulations, and experimental testing of various structural forms and heat exchange scenarios. Additionally, the integration of advanced thermal insulation materials and low-energy technologies will be explored to further enhance energy efficiency.

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