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NUMERICAL SIMULATION OF SEPARATED FLOWS BASED ON NON-STATIONARY NAVIE-STOKES EQUATIONS

Rajabov Azamat

National University of Uzbekistan

Abstract

The fluid flow in a two-dimensional channel with a sudden expansion (x/h = 2) is studied. The calculations were performed for the laminar flow regime based on the numerical integration of the nonstationary Navier-Stokes equations. Various flow characteristics are determined at Re = 100–800. Results are obtained for longitudinal velocity profiles for various sections of the channel. The present work is devoted to modeling the separation flow around a plate in a cylinder. Mathematical modeling of the fluid flow and the results of a computational study of the put forward hypothesis are carried out using the example of the fluid flow supply to the cylinders. The unsteady Reynolds-averaged Navier-Stokes equations are used as a mathematical model of flows. For the numerical solution of the presented systems of equations, the control volume method and the SIMPLE algorithm were used.

Keywords: mathematical model, laminar flow, non-stationary Navier-Stokes equations, numerical solution, difference scheme.

Introduction.

In modern conditions of the development of road transport, the oil and gas industry and other industries, it became necessary to conduct research on a wider class of different tasks that differ in features. To solve scientific, technological and environmental problems in these industries, the development of new mathematical models is required [1-5].

A necessary step in the development of internal combustion engines and their elements is the numerical simulation of complex hydrodynamic flows. These flows arise when a liquid or gas flows around bodies of various shapes with arbitrary initial and boundary conditions for the region under consideration. Therefore, an urgent task at present is the development of numerical methods for solving systems of equations of unsteady flows based on finite-difference schemes of a higher order of accuracy [6-12].

The first analytical study of the laminar separation of stationary two-dimensional incompressible fluid flows in straight channels was carried out in 1910 by P.R.G. Blasius [1]. In view of the great practical importance of such flows, theoretical and experimental studies of both laminar [2–4] and turbulent [5–7] flow regimes of incompressible and compressible fluids have been carried out. In most works, channels with symmetric expansion are considered [8–14]. Thus, in [8, 9], the results of an experimental study of the formation of a circulation zone behind a bar are presented. Separated flows are described using the equations of motion in the boundary layer approximation [14, 15]. A significant and important amount of work on the calculation of flows in suddenly expanding channels was experimentally performed in [4]. Numerical simulation of such a problem was carried out in [16]. The authors of [17] used the theory of potential flow to obtain a solution to this problem. However, the theory of potential flow failed to predict the zones of separation and connection of flows. The initial numerical predictions of the behavior of flows behind the backward step were made by the authors of. The size of the recirculation zone behind the step was predicted in [18].

This paper presents a numerical method for calculating laminar flows of a viscous fluid in a cylinder with a subsonic flow. Reynolds-averaged unsteady Navier-Stokes equations written with respect to Cartesian coordinates are used as a mathematical model of flows [12-18]. This article deals with laminar flow. Therefore, Reynolds number 200-800 is obtained.

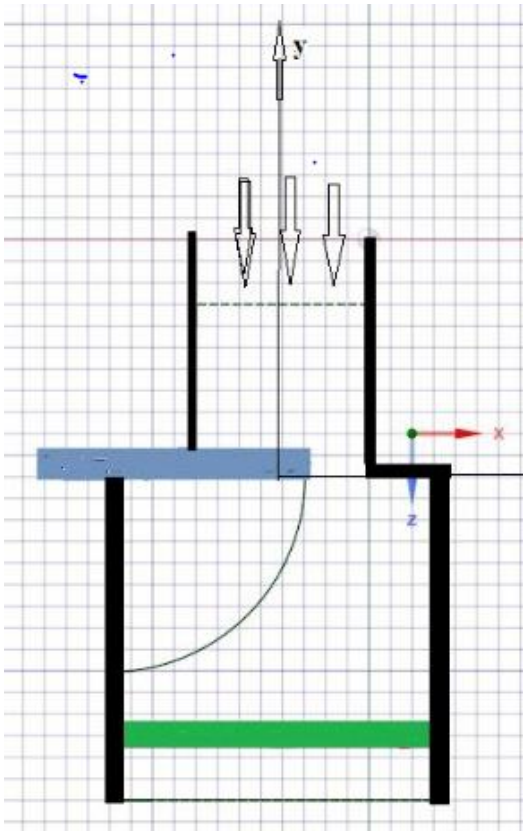
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Physical and mathematical formulation of the problem.

A two-dimensional laminar fluid flow in a flat channel with width *h* is considered .

The channel length is 5*h* , where *h* is the channel width. This length is chosen to form a fully developed laminar flow. The physical picture of the analyzed flow in a two-dimensional channel is one of the simplest configurations of the computational domain (Fig. 1).



Rice. 1. Scheme of the computational domain in a flat channel

The system of non-stationary Reynolds-averaged Navier–Stokes equations after using the Businesk hypothesis and the continuity equation with a constant density in Cartesian coordinates has the following form [19]:

$$\begin{cases} \dfrac{\partial U}{\partial x}+\dfrac{\partial V}{\partial y}=0 \\ \dfrac{\partial U}{\partial t}+U\dfrac{\partial U}{\partial x}+V\dfrac{\partial U}{\partial y}+\dfrac{\partial p}{\rho\partial x}=\dfrac{1}{\mathrm{Re}}\left(\dfrac{\partial^2 U}{\partial x^2}+\dfrac{\partial^2 U}{\partial^2 y}\right), \\ \dfrac{\partial U}{\partial t}+U\dfrac{\partial V}{\partial x}+V\dfrac{\partial V}{\partial y}+\dfrac{\partial p}{\rho\partial y}=\dfrac{1}{\mathrm{Re}}\left(\dfrac{\partial^2 V}{\partial x^2}+\dfrac{\partial^2 V}{\partial^2 y}\right). \end{cases} \quad (1)$$

Here U,V - respectively longitudinal, vertical components of the flow velocity vector, p is the hydrostatic pressure, $\mathrm{Re}=HU_0/\nu$ is the Reynolds number. Re - Reynolds number. On all fixed solid walls, the obvious no-slip boundary conditions are set $U\big|_\Gamma=0$ and $V\big|_\Gamma=0$, where Γ is a solid boundary. U_0 – inlet velocity

On all fixed solid walls, we have the following no-slip boundary conditions

$$U\big|_\Gamma=0 \qquad \text{and} \qquad V\big|_\Gamma=0,$$

where Γ is a hard boundary.

Solution method.

The numerical solution of the presented systems of equations was carried out in the physical variables velocity - pressure by physical splitting of the velocity and pressure fields. In this case, for the transfer equations, a chess difference grid was used by the control volume method [20]:

$$\begin{cases} \dfrac{\tilde{U}_{i,j}-U^n_{i,j}}{\Delta t}+U^n_{i,j}\dfrac{\partial U^n_{i,j}}{\partial x}+V^n_{i,j}\dfrac{\partial U^n_{i,j}}{\partial y}+\dfrac{\partial p^n_{i,j}}{\partial x}=\dfrac{1}{\mathrm{Re}}\left(\dfrac{\partial^2\tilde{U}_{i,j}}{\partial y^2}+\dfrac{\partial^2 U^n_{i,j}}{\partial x^2}\right), \\ \dfrac{\tilde{V}_{i,j}-V^n_{i,j}}{\Delta t}+U^n_{i,j}\dfrac{\partial V^n_{i,j}}{\partial x}+V^n_{i,j}\dfrac{\partial V^n_{i,j}}{\partial y}+\dfrac{\partial p^n{}_{i,j}}{\partial y}=\dfrac{1}{\mathrm{Re}}\left(\dfrac{\partial^2\tilde{V}_{i,j}}{\partial y^2}+\dfrac{\partial^2 V^n_{i,j}}{\partial x^2}\right). \end{cases} \quad (2)$$

$$\begin{cases} U^{n+1}=\tilde{U}-\Delta t\dfrac{\partial\delta p}{\partial x} \\ V^{n+1}=\tilde{V}-\Delta t\dfrac{\partial\delta p}{\partial y} \end{cases} \quad (3)$$

Equations (2) superscript "~ $\tilde{U}$" denotes an intermediate grid function for the velocity vector; $\delta p=p^{n+1}-p^n$ pressure correction. Multiplying equation (2) by the gradient and taking into account the solenoidality of the velocity vector at $(n+1)$the th time layer, we obtain the Poisson equation for determining the correction to pressure:

$$\Delta t\left(\frac{\partial^2\delta p}{\partial x^2}+\frac{\partial^2\delta p}{\partial y^2}\right)=\frac{\partial\tilde{U}}{\partial x}+\frac{\partial\tilde{V}}{\partial y}. \quad (4)$$

The solution of equation (4) was carried out by the iteration method, for which equation (4) was reduced to a parabolic form

$$\frac{\partial\delta p}{\partial t_0}-\Delta t\left(\frac{\partial^2\delta p}{\partial x^2}+\frac{\partial^2\delta p}{\partial y^2}\right)=\frac{\partial\tilde{U}}{\partial x}+\frac{\partial\tilde{V}}{\partial y}. \quad (5)$$

where fictitious time t_0 is an iterative parameter. When solving equation (5) for the time step, one can write a $\Delta t_0 = a_1 \Delta t$, constant value , as a rule, less than unity and is chosen from the condition of rapid convergence of the numerical process. a_1 The Neumann condition is used as the boundary condition for the pressure correction, which is satisfied if $\tilde{U}$ the exact value of is used at the boundary U^{n+1}. For the numerical solution of the transport equation of system (2), a finite-difference upwind scheme for the convective terms is used. And the diffusion terms were approximated by the central difference implicitly.

The upstream scheme has the form [21–22]

$$\Phi = \begin{bmatrix} U \\ V \end{bmatrix}, \quad UU = \begin{bmatrix} U_{i,j}^n \\ 0.25\left(U_{i,j}^n + U_{i-1,j}^n + U_{i,j+1}^n + U_{i-1,j+1}^n\right) \end{bmatrix},$$

$$VV = \begin{bmatrix} 0.25\left(V_{i,j}^n + V_{i,j-1}^n + V_{i+1,j-1}^n + V_{i+1,j}^n\right) \\ V_{i,j}^n \end{bmatrix}, \quad w = \begin{bmatrix} \frac{\partial p}{\rho \partial x} \\ \frac{\partial p}{\rho \partial y} \end{bmatrix}$$

$$\frac{\Phi_{i,j}^{n+1} - \Phi_{i,j}^n}{\Delta t} + 0.5\left(UU + |UU|\right)\frac{\Phi_{i,j}^n - \Phi_{i-1,j}^n}{\Delta x} + 0.5\left(UU - |UU|\right)\frac{\Phi_{i+1,j}^n - \Phi_{i,j}^n}{\Delta x} +$$
$$+0.5\left(VV + |VV|\right)\frac{\Phi_{i,j}^n - \Phi_{i,j-1}^n}{\Delta y} + 0.5\left(VV - |VV|\right)\frac{\Phi_{i,j+1}^n - \Phi_{i,j}^n}{\Delta y} =$$
$$= \frac{\Phi_{i+1,j}^n - 2\Phi_{i,j}^n + \Phi_{i-1,j}^n}{\mathrm{Re}\,\Delta x^2} + \frac{\Phi_{i,j+1}^{n+1} - 2\Phi_{i,j}^{n+1} + \Phi_{i,j-1}^{n+1}}{\mathrm{Re}\,\Delta y^2} - w. \qquad (6)$$

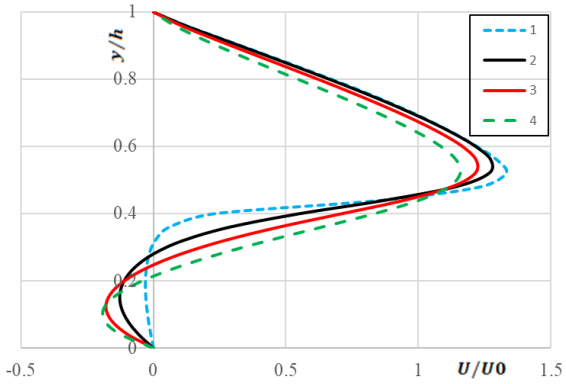
For the numerical solution of equation (5), a semi-implicit scheme is used, which is effectively implemented by the sweep method

$$\frac{\delta p_{i,j}^{n+1} - \delta p_{i,j}^n}{\Delta t_0} - \left(\frac{\delta p_{i+1,j}^n - 2\delta p_{i,j}^{n+1} + \delta p_{i-1,j}^n}{\Delta x^2}\right) - \left(\frac{\delta p_{i,j+1}^{n+1} - 2\delta p_{i,j}^{n+1} + \delta p_{i,j-1}^{n+1}}{\Delta y^2}\right) =$$
$$= \frac{1}{\Delta t}\left(\frac{\tilde{U}_{i,j} - \tilde{U}_{i-1,j}}{\Delta x} + \frac{\tilde{V}_{i,j} - \tilde{V}_{i,j-1}}{\Delta y}\right). \qquad (7)$$

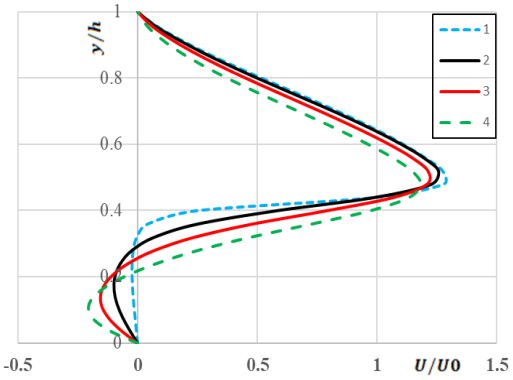
Thus, first, the system of equations (2) , (3) is solved by the establishment method , then equation (7) and, in accordance with (5), the velocity vector on the th time layer and pressure $p^{n+1} = p^n + \delta p$ are determined $(n+1)$. It is easy to show that the calculation scheme has an order of accuracy $O(\Delta t, \Delta x, \Delta y^2)$. This means that the velocity and pressure components are determined at different nodes. This approach is similar to the SIMPLE methods and provides certain advantages in calculating the pressure field [23]. The arrangement of cells and nodes is similar to that of the SIMPLE method. The article we used grid 100x100.

Results.

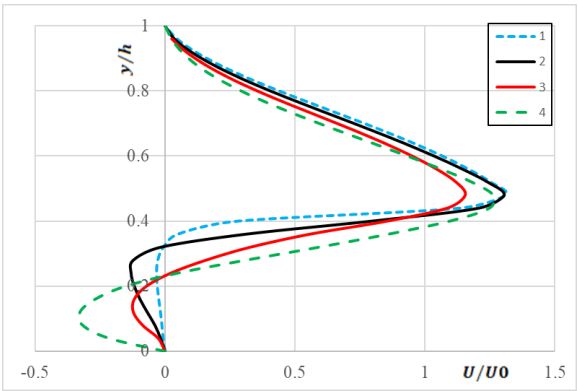
Figure 3 shows graphs of comparison of calculated data. The figures show the profiles of the longitudinal velocity U in various measured sections at distances from the entrance to a wide channel and grid tension. Changing the set does not affect the result.



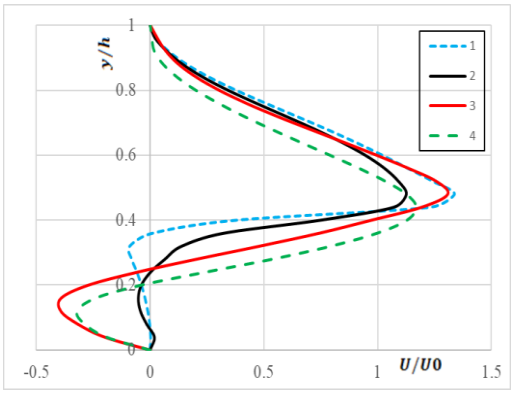
a)



b)



c)



e)

Pic. 2.

1) $\frac{x}{H} = 2$; 2) $\frac{x}{H} = 4$; 3) $\frac{x}{H} = 6$; 4) $\frac{x}{H} = 6$.

a) – Re =200; b) – Re=400; c) –Re = 600; e) – Re = 800.

Since there are no experimental results, the graphs are not compared with the experimental results. As can be seen from the graphs, clusters have formed at the place of sharp expansion. Therefore, the obtained results are reliable.

Conclusions.

The article presents the effectiveness of joint use of the well-known finite-difference scheme Against the flow with the SIMPLE procedure for studying the processes of flow separation. As a result of a detailed numerical study of velocity fields in a flat channel with a rectangular ledge, including secondary vortex formations and unsteady separated flow regimes. As it can be seen from the graphs, when the Reynolds number is not equal to 200, piles are not formed in the expansion part of the channel. When the Reynolds number is greater than 200, it can be seen from the graphs that clumps are formed in the expansion part of the channel.

References

1. Blasius H. Laminare Stromung in Kanalen Wecselnder Briete , Zeitschrift filer Mathematik und Physik. 10 . p. 225. (1910)
2. The starting flow down a step , J. Fluid Mech, 69(2).pp. 229-240. (1975)
3. Sinha SP, Gupta AK, Oberai M. M . Laminar tear-off flow around ledges and caverns , Rocket technology and cosmonautics, 19 (12), pp. 33-37.(1981)
4. Armaly B. F., Durst F., Pereira J. C. F., Schonung B. Experimental and theoretical investigation of backward-facing stap flow , J. Fluid Mech, 127. pp. 473-496.(1983
5. Zheng P. Detached currents. - M.: Mir. 2.P,354. (1973)
6. Honji H. The starting flow down a step. J. Fluid Mech., 1975, vol. 69, no. 2, pp. 229– 240. DOI: https://doi.org/10.1017/S0022112075001413
7. Синха С.П., Гупта А.К., Оберай М.М. Ламинарное отрывное обтекание уступов и каверн. Ч. 1. Течение за уступом. Ракетная техника и космонавтика, 1981, т. 19, № 12, с. 33–37.
8. Чжен П. Отрывные течения, М., Мир, 1972.
9. Гогиш Л.В., Степанов Г.Ю. Турбулентные отрывные течения. М., Наука, 1979.
10. Le H., Moin P., Kim J. Direct numerical simulation of turbulent flow over a backward-facing step. J. Fluid Mech., 1997, vol. 330, pp. 349–374. DOI: https://doi.org/10.1017/S0022112096003941
11. Durst F., Melling A., Whitelow J.H. Low Reynolds number flow over a plane symmetric sudden expansion. J. Fluid Mech., 1974, vol. 64, iss. 1, pp. 111–118. DOI: https://doi.org/10.1017/S0022112074002035
12. Cherdron W., Durst F., Whitelow J.H. Asymmetric flows and instabilities in symmetric ducts with sudden expansions. J. Fluid Mech., 1978, vol. 84, iss. 1, pp. 13– 31. DOI: https://doi.org/10.1017/S0022112078000026
13. Macadno E.O., Hung T.-K. Computational and experimental study of a captive annular eddy. J. Fluid Mech., 1967, vol. 28, iss. 1, pp. 43–64. DOI: https://doi.org/10.1017/S0022112067001892
14. Kumar A., Yajnik K.S. Internal separated flows at large Reynolds number. J. Fluid Mech., 1980, vol. 97, iss. 1, pp. 27–51. DOI: https://doi.org/10.1017/S0022112080002418
15. Плоткин А. Расчеты спектральным методом некоторых отрывных ламинарных течений в каналах. Аэрокосмическая техника, 1983, № 7, с. 75–85.
16. Acrivos A., Schrader M.L. Steady flow in a sudden expansion at high Reynolds numbers. Phys. Fluids, 1982, vol. 25, iss. 6, pp. 923–930. DOI: https://doi.org/10.1063/1.863844
17. Куон О., Плетчер Р., Льюис Дж. Расчет течений с внезапным расширением при помощи уравнений пограничного слоя. Теор. основы инж. расч., 1984, т. 106, № 3, с. 116–123.
18. Плетчер Л. Пределы применимости уравнений пограничного слоя для расчета ламинарных течений с симметричным внезапным расширением. Теор. основы инж. расч., 1986, № 2, с. 284–294.
19. Loitsyansky L.G. Mechanics of liquid and gas. - M.: Nauka, 1987. - 678 p.
20. Patankar SV Numerical Heat Transfer and Fluid Flow. Taylor and Francis. ISBN 978-0-89116-522-4, 1980.
21. Samarsky A.A. Theory of difference schemes.-M.: Nauka, 1983.-656p.
22. Anderson D., Tannehill J., Pletcher R. Computational fluid mechanics and heat transfer : In 2 vols. T. 1: Per. from English. - M.: Mir, 1990. - 384 p.
23. Madaliev E, Madaliev M, Adilov K, and Pulatov T. Comparison of turbulence models for two-phase flow in a centrifugal separator in E3S Web of Conferences. 264 . 1009. (2021). https://doi.org/10.1051/e3sconf/202126401009