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MATHEMATICAL MODEL FOR THE DYNAMIC EQUATION OF A TRUCK'S MOTION

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Abstract

This paper addresses the problem of formulating the dynamic equations of motion for a truck and developing its mathematical models. The motion of the truck is mathematically represented based on Newton's second law and Lagrange's equations. The dynamic characteristics of the truck, including its speed, force, and mass dependence, are analyzed using Newton's second law, while Lagrange's equations are applied to calculate the system's kinetic and potential energies. Using these methods, differential equations are formulated for the truck's motion, and its motion trajectories are analyzed. The study focuses on the dynamics of a truck moving on an inclined plane, calculating the motion trajectories at different speeds and angles. These calculations reveal the relationships between the truck's maximum speed, energy consumption, and motion trajectories. Furthermore, the energy accumulation process for trucks of various masses is examined, providing insights into how energy is consumed and accumulated during motion. Based on the results, potential ways to optimize the truck's motion and reduce energy consumption are discussed. The scientific significance of this study lies in its potential to create opportunities for efficient and energy-saving motion, particularly in the fields of transportation and mechanical systems. This mathematical model is also useful for improving energy processes in transportation systems and developing practical solutions to enhance the operational efficiency of trucks.

Keywords: Mathematical model, Newton's second law, Lagrange's equation, trajectory, mass, kinetic energy, dynamic model, angle, speed, braking.

1. Introduction

The modeling of a wheeled vehicle starts with the formulation of the basic dynamics of the vehicle. The motion of the vehicle can be mathematically described based on the fundamental laws of physics. Describing a complex system like a vehicle with lower subsystems requires the development of a complex mechanical and mathematical model. Dynamic models allow for a detailed analysis and prediction of the braking process, considering factors such as braking force, vehicle mass, speed, and other variables. This, in turn, enables effective and safe braking management. Using dynamic models, the impact of changing conditions (e.g., road conditions, load, and speed) on the braking system can be identified. This helps enhance safety during the braking process, as control systems adjust according to varying conditions. The use of dynamic models in the braking process aids in optimizing system speed and reaction time, ensuring that braking is carried out quickly and efficiently. Through dynamic models, the regenerative braking process can be improved, reducing energy consumption and increasing energy recovery potential. Overall, the use of dynamic models plays a crucial role in making braking control more effective, safe, and energy-efficient [1-5].

When a truck descends a slope with the brakes applied, its energy recovery efficiency increases. In this system, the kinetic energy released during braking is converted into potential energy, which is then stored in the recuperator device. The system consists of power converters that are closely monitored by the generator and the central control unit. Since safety and driving comfort are heavily dependent on braking, particularly in uncertain road conditions, high-quality electric brake control methods are essential. To design and develop control techniques, the dynamic models of the braking force and load acting on the vehicle are of significant importance.

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2. Mathematical Formulation of the Problem

For this, we first present the dynamic model of the truck when it moves in a straight line on a flat surface. In modern vehicles, the mass of the transported load is typically comparable to the mass of the vehicle itself. Therefore, when the transported load has the potential for relative motion within the vehicle body, considering both the vehicle and the transported load system together allows for accurate dynamic calculations. The aim of this work is to develop the mathematical model for a stationary vehicle. The calculation scheme is presented in Figure 1. The dynamic equations of motion for the vehicle's suspension system are as follows.

$$m\left(\dot{V}_y + V_x\omega\right) = F_{1n}\cos\left(\delta_n(t)\right) + F_{1m}\cos\left(\delta_m(t)\right) + \\ + F_{2n} + F_{2m} \quad (1)$$

$$I_z\dot{\omega} = a\left[F_{1n}\cos\left(\delta_n(t)\right) + F_{1m}\cos\left(\delta_m(t)\right)\right] - b[F_{2n} + \\ + F_{2m}] \quad (2)$$

$$I_x\ddot{\varphi} = -(K_1 + K_2)\varphi - (\sigma_1 + \sigma_2)\dot{\varphi} + M_1 + M_2 \quad (3)$$

The first equation describes the motion along the horizontal y-axis, while the second and third equations describe the rotation of the chassis around the z and x axes, respectively. In equations (1-3), the following symbols are used: m - vehicle mass; a and b - dimensions defining the position of the vehicle's center of gravity; L - vehicle wheelbase; F - lateral force acting on the wheels; P - normal force between the wheel and the road; V_y and V_x - projections of the velocity of the mass center along the coordinate axes; δ - acceleration describing the deviation from linear motion; M - moments of centrifugal forces; I_x and I_z - moments of inertia of the vehicle relative to the x and z axes, respectively; k_x and k_z - stiffness coefficients; c_x and c_z - damping coefficients; θ_w-average wheel rotation angle.

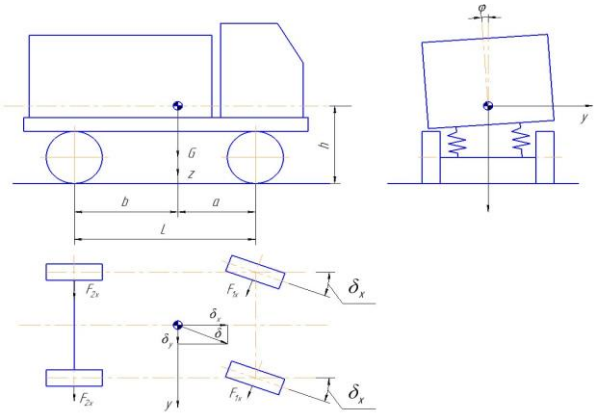


Figure 1. Calculation Scheme of the Vehicle

Indices 1 and 2 correspond to the front and rear axles, while indices n and m correspond to the right and left wheels, respectively. The forces acting on the vehicle's wheels can be calculated using formulas from [6] for the normal forces and the wheel slip angles between the wheel and the road.

The normal pressure forces of each vehicle wheel on the road surface can be determined using equations derived through kinematical methods:

$$\begin{cases} P_{1n} = \frac{Pb}{a+b}\cdot\frac{c_1+e_1\sin\varphi}{2c_1} + T_1; \\ P_{1m} = \frac{Pb}{a+b}\cdot\frac{c_1-e_1\sin\varphi}{2c_1} - T_1; \\ P_{2n} = \frac{Pa}{a+b}\cdot\frac{c_2+e_2\sin\varphi}{2c_2} + T_2; \\ P_{2m} = \frac{Pa}{a+b}\cdot\frac{c_2-e_2\sin\varphi}{2c_2} - T_2; \end{cases} \quad (4)$$

Here, T1 and T2 are the components of the centrifugal forces applied to the front and rear axles, which are determined from the relationships of the moments:

$$\begin{cases} T_1 = \frac{\Phi b}{a+b}\cdot\frac{h_1}{2c_1} + \frac{K_1\varphi+\sigma_1\sin\varphi}{2c_1}; \\ T_2 = \frac{\Phi b}{a+b}\cdot\frac{h_2}{2c_2} + \frac{K_2\varphi+\sigma_2\sin\varphi}{2c_2}, \end{cases} \quad (5)$$

Here, ϕ - represents the result of the centrifugal inertia forces.

$$\Phi = m(V_x\omega)$$

he moments of the centrifugal forces that occur at the front and rear axles of the vehicle are expressed through the result of the centrifugal inertia forces, respectively.

$$M_1 = \frac{\Phi b}{a+b}e_1,$$

$$M_1 = \frac{\Phi b}{a+b}e_2;$$

$$e_1 = h - h_1,$$

$$e_2 = h - h_2$$

Here: ϕ - is the vehicle body's rotation angle relative to the longitudinal axis; lf and lr - are the front and rear half-lengths of the vehicle, respectively; h - is the dimension that defines the position of the vehicle's center of gravity relative to the road surface; lf1 and lr1- are the dimensions that define the position of the mass center of the front and rear axles relative to the road surface, respectively; Mf and Mr - are the centrifugal moments generated at the front and rear axles of the vehicle, respectively.

For each vehicle wheel, the slip angles can be determined from the following relationships:

$$\begin{cases} \tau_{1n} = \gamma_{1n} + \beta_{1n} + \delta_n - arctg\left(\frac{V_y+\omega a}{V_x-\omega c_1}\right); \\ \tau_{1m} = \gamma_{1m} + \beta_{1m} + \delta_m - arctg\left(\frac{V_y+\omega a}{V_x+\omega c_1}\right); \\ \tau_{2n} = \gamma_{2n} + \beta_{2n} - arctg\left(\frac{V_y-\omega b}{V_x-\omega c_2}\right); \\ \tau_{2m} = \gamma_{2m} + \beta_{2m} - arctg\left(\frac{V_y-\omega b}{V_x+\omega c_2}\right); \end{cases} \quad (6)$$

These are $\gamma, \beta -$ the angles defined by the alignment («развалом») and convergence «схождением» of the vehicle wheels, respectively. These calculation formulas are used when the truck moves in a straight line [7]. The calculation results are presented in [8, 9].

Now, we present the motion law of the truck moving on an inclined plane. It is known that during braking, the dynamic model of the truck is studied in detail in [10]. The mass of the vehicle changes depending on the load, so accurate estimation of the mass is required. Electromechanical brake systems have been introduced to improve the safety and energy efficiency of vehicles. These systems replace hydraulic brake components with electronics, thereby simplifying the brake system. Although this increases the system's flexibility, it requires the development of special control strategies. The model of the electromechanical brake system is presented in [11]. Along with the hydraulic brake systems present in vehicles, the distribution of braking force is discussed in [12, 13], and the optimal distribution of braking moment between electrical and friction brakes is analyzed in [14].

The dynamic model of the vehicle during braking can be seen in the following Figure 2. It is influenced by the friction force, normal force, and gravitational force, and is utilized as follows.

For trucks moving on inclined roads, along the x-axis, the acceleration caused by the gravitational force, the weight force (mg sin α), and the frictional force arise, where α is the road's slope angle. Along the y-axis, the acceleration caused by the gravitational force is zero, and the weight force (mg cos α) and the normal force arise. For this, we use Newton's second law. It can be written in vector form as for:

$$m\vec{a} = \sum \vec{F} \quad (7)$$

Newton's second law can be written with respect to the axes as follows.

$$\begin{cases} m\ddot{x} = \sum F_x \\ m\ddot{y} = \sum F_y \end{cases} \Rightarrow \begin{cases} m\ddot{x} = (mg)_x + F_{friction} \\ m\ddot{y} = (mg)_x + N \end{cases} \quad (8)$$

Based on Figure 2, equation (8) can be written as follows.

$$\begin{cases} m\ddot{x} = mg\sin\alpha - \mu N \\ 0 = -mg\cos\alpha + N \end{cases} \quad (9)$$

Here, the frictional force is equal to the following.

$$F_{ish} = -\mu N$$

From the second equation of (9), we can find the normal force. $N = mg\cos\alpha$ Then, the general form of (9) will be equal to the following.

$$m\ddot{x} = mg\sin\alpha - \mu(mg\cos\alpha) \Rightarrow \ddot{x} = g(\sin\alpha - \mu\cos\alpha) \quad (10)$$

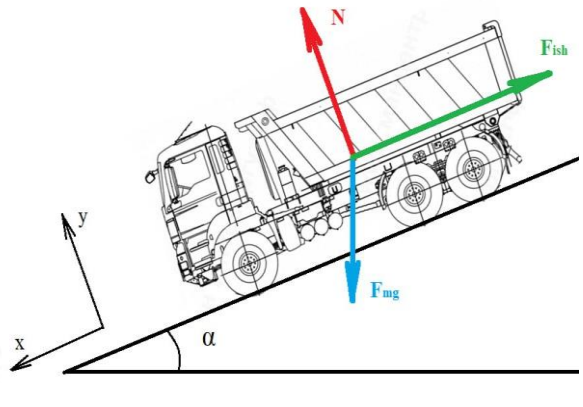


Figure 2. The dynamic model of the vehicle during braking

To calculate equation (10), we use the following initial conditions.

$$t = 0: x = 0;$$
$$t = 0: \dot{x} = \frac{d\vartheta}{dt} = \vartheta_0 \quad (11)$$

By integrating equation (10) twice, we obtain the following.

$$x = \frac{gt^2}{2}(\sin\alpha - \mu\cos\alpha) + C_1 t + C_2 \quad (12)$$

To find the constants in equation (12), we use equation (11) and obtain the following result.

$$x = \frac{gt^2}{2}(\sin\alpha - \mu\cos\alpha) + \vartheta_0 t \quad (13)$$

During braking in trucks, the kinetic energy is converted into potential energy, and it is equal to the following.

$$E_p = mgh \quad (14)$$

3. Analysis of the Results

Using equations (13) and (14), we obtain the following results. In Figure 3, the trajectories of speeds at different angles are presented. Here, the initial speed is 20 m/s.

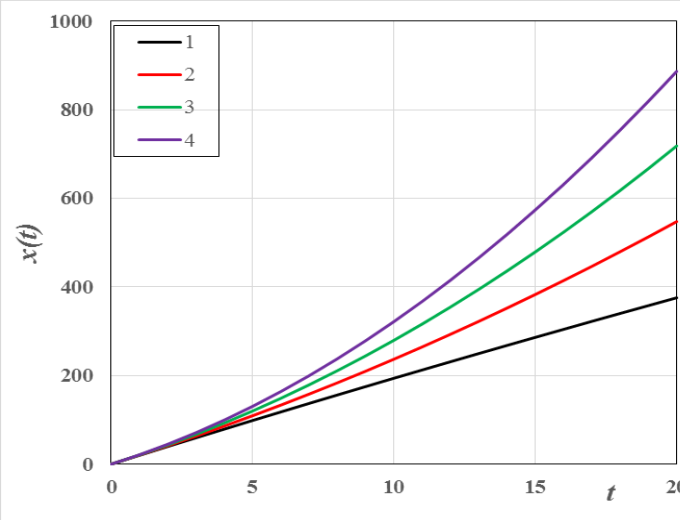
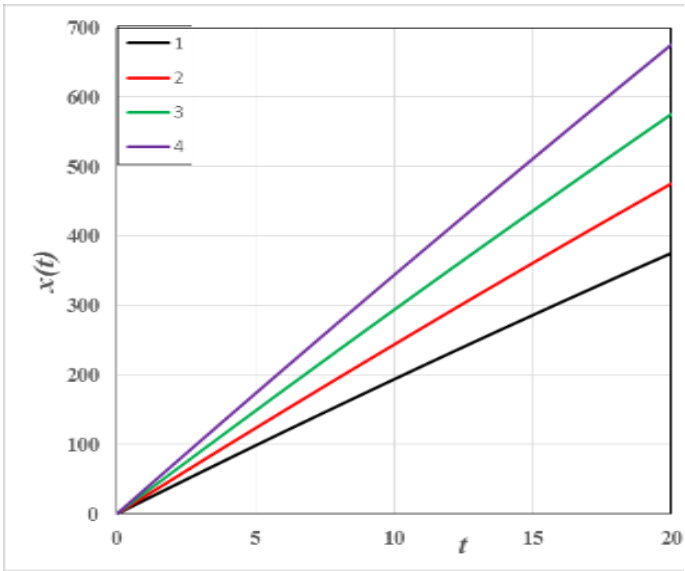
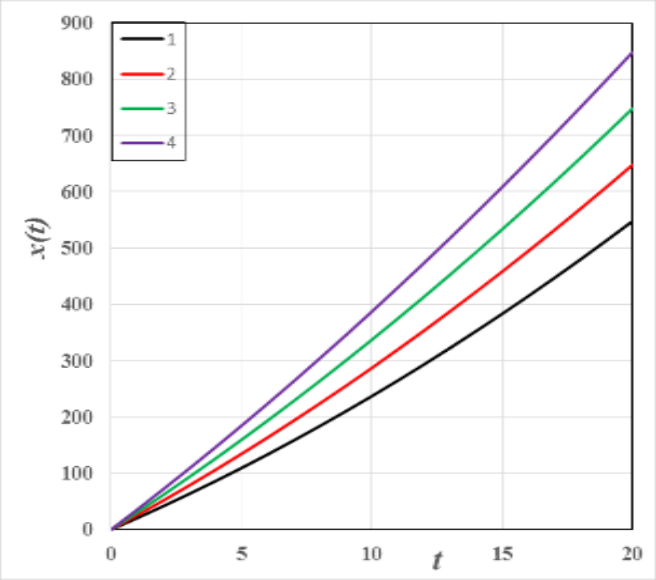


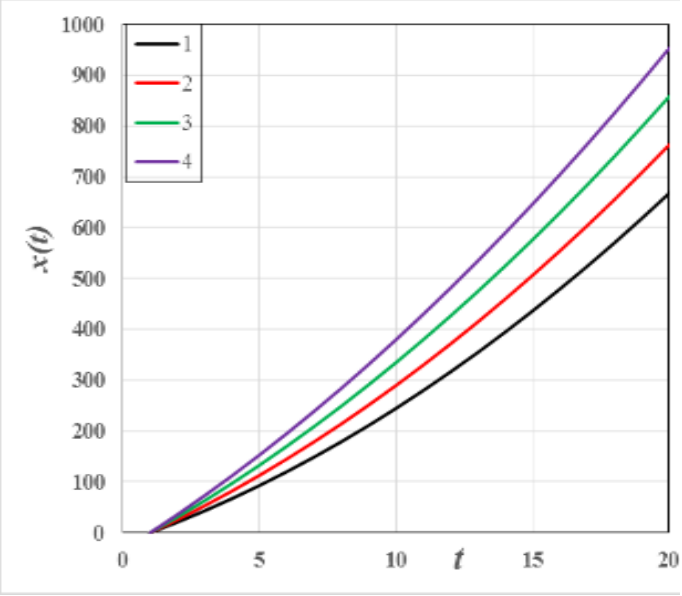
Figure 3. The graph of the motion trajectory for different speeds. 1-50, 2-100, 3-150, 4-200



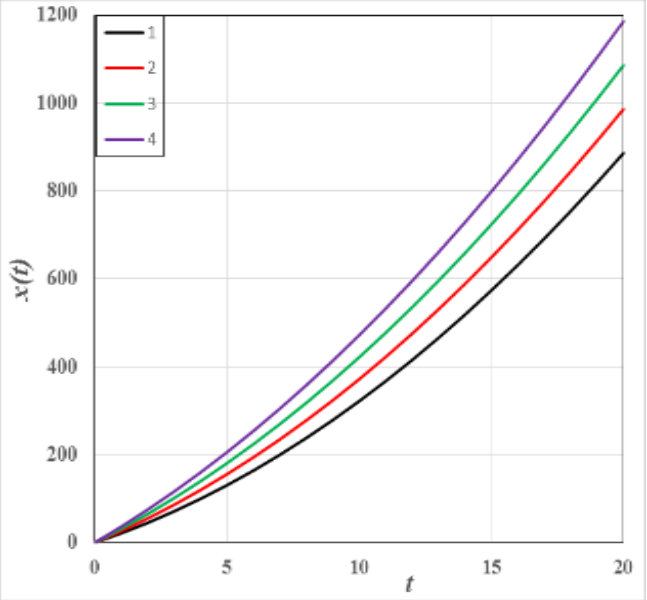
$\alpha = 5^0$



$\alpha = 10^0$



$\alpha = 15^0$



$\alpha = 20^0$

Figure 4. The motion trajectory for different-speeds: 1-20 m/s, 2-25 m/s, 3-30 m/s, 4-35 m/s

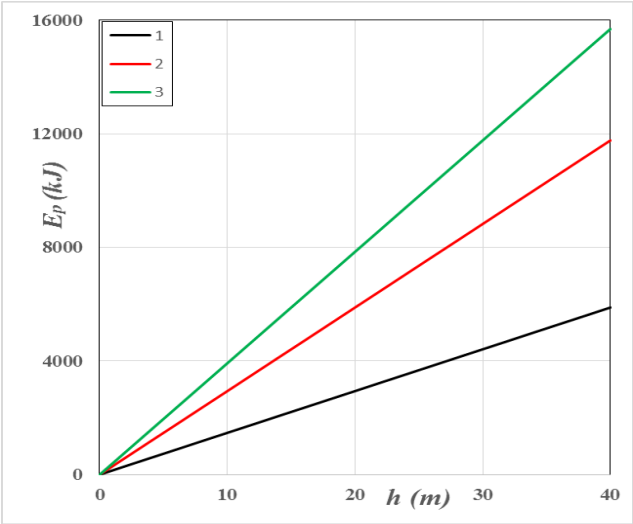


Figure 5. Change in potential energy with respect to height. 1-15 t, 2-28 t, 3-40 t

In Figure 5, the graphs of potential energy with respect to height are presented for the truck in three different loading conditions: with its own weight (15 t), half-loaded (28 t), and fully-loaded (40 t).

3. Conclusions

This study focuses on developing mathematical models for the dynamic motion of trucks and analyzing their behavior on both flat and inclined planes. Dynamic equations describing the truck's motion were formulated based on Newton's second law and Lagrange's equations, and the motion trajectories of the truck on an inclined plane were calculated. The results show that, for a constant speed, as the incline angle increases, the distance traveled by the truck also increases. This finding helps to deepen the understanding of the influence of the incline angle on the motion trajectory. In cases with varying speeds, the truck's trajectory at higher speeds extends more, meaning the motion covers a larger distance at higher speeds. This also highlights key aspects related to energy consumption and the efficiency of the dynamics. The variation in speed also alters the truck's energy accumulation process: at higher speeds, more energy is consumed, but the truck covers a longer distance. The study also found that the truck accumulates more energy when fully loaded. In this case, the truck's increased mass leads to higher kinetic energy, providing better control over the motion and energy consumption. The dynamic analyses and results presented in this research have significant scientific and practical implications for improving energy efficiency and optimizing motion in trucks. The findings also offer insight into how trucks behave under various speeds and angles, aiding in the development of new approaches for efficient management in transportation systems. Using these models, practical measures can be developed in the future to optimize energy consumption and improve motion efficiency of trucks.

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