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MATHEMATICAL MODELING OF TURBULENT FLOWS OF SUSPENSIONS IN HORIZONTAL PIPELINES

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Abstract

The paper presents a numerical study of multiphase fluid flows. Numerical study of turbulent flows of fluid with particles for arbitrary concentrations. The model includes equations for a two-phase flow as a whole with rheological relations and an equation for the transfer of particle concentration taking into account interphase slip. The statistical model of turbulence takes into account the modulation of turbulence by particles. The model has been tested on problems of steady flow with heavy particles in a horizontal pipe. Transporting solids as a water carrier through long pipelines is widely used in many industries and power plants. Transporting slurry through a vertical pipeline is a challenging task and requires modification to overcome pressure loss and power consumption requirements. In this regard, a numerical simulation of a three-dimensional horizontal slurry pipeline carrying glass beads with spherical particles with a diameter of 440 μm and a density of 2650 kg/m3 was carried out. The simulation is performed using an Eulerian multiphase model with SST turbulence model in the solid concentration range of 10–20% (by volume) for mean flow velocities of 1–4 ms–1. A parametric study is conducted to visualize and understand the transition flow. Finally, solid concentration contour, velocity contour, solid concentration profiles, velocity profiles results and predictions for slurry pipelines.

Keywords: suspension, turbulence, pipe flow, numerical simulation.

1. Introduction

Liquid flows carrying solid particles are observed in a number of natural phenomena and in technical devices. The study of particle motion in a turbulent flow and their influence on the characteristics of the carrier medium is one of the important and difficult problems of multiphase flow mechanics.

Currently, the approach of direct numerical modeling of suspension flows with resolution of liquid flow around individual particles has been rapidly developed [1]. However, this approach is too expensive for solving a wide range of engineering problems. The most suitable models of turbulent suspension flows are still two-fluid models describing the movement of the carrier phase and the movement of a group of particles, based on a continuum representation and using Reynolds or Favre-averaged equations.

When constructing a turbulence model for a particle-liquid system, it is convenient to distinguish three levels of description: microscopic, mesoscale, and macroscale [2]. At the microscopic level, the motion of individual particles and the flow of fluid flowing around them are considered. The mesoscale level corresponds to the averaged description of the chaotic or pseudo-turbulent motion of particles caused by interparticle interaction. The macroscale description considers the patterns of turbulent motion of fluid and the involvement of particles in pulsating motion by turbulent vortices.

Hydrodynamic interactions between particles and direct collisions between them lead to the appearance of pseudo-turbulent pulsations and stresses of the particle field, which must be taken into account when modeling flows with a high particle concentration. The description of stresses in the particle phase caused by inter particle interactions can be performed within the framework of the kinetic model of granular media or using a phenomenological rheological model. The construction of a kinetic model of granular media is performed by analogy with the kinetic theory of molecular gas motion. The evolution of pulsations is

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described by the pseudo-temperature transfer equation taking into account the generation of chaotic motion in a gradient flow, dissipation of pulsation energy due to viscous losses and inelastic collisions. At present, the kinetic model is considered well tested and suitable for the mesoscale description of flows of moderately dense granular media.

In mathematical models of suspension flows, phenomenological algebraic dependences of normal and shear stresses on particle concentration and velocity gradients are introduced [2, 3]. Depending on the local Stokes number, particle Reynolds number and particle concentration, the following particle motion regimes in the flow are distinguished: free fall and prolonged interparticle contact, inertial and viscous regimes. Kinetic or rheological approaches to describing mesoscale flow can be used to construct a model of macroscale turbulent flow [4–8].

Most numerical studies of the class of flows under consideration have been performed using two-fluid models that include equations of motion for each phase. However, for the vast majority of practical purposes, solving the full system of a two-velocity model is not only computationally expensive, but also unnecessary. Steady-state or weakly time-varying suspension flows are characterized by the smallness of the dynamic relaxation time of particles compared to the hydrodynamic time. In this case, the full two-velocity model can be reduced to a model with a quasi-equilibrium velocity of interphase slip [9–11].

Highly concentrated fluid flows with relatively small particles, for which the Stokes number does not exceed 10, can be considered at the mesoscale level as suspension flows with some rheological law describing the total stresses in the mixture. Mesoscale stresses in the liquid phase include the stress that is the contribution of particles to the hydrodynamic part of the stresses of the mixture as a whole [12]. As a result, the decomposition of the mixture stresses into stress components in the liquid and solid phases, which is necessary for constructing two-fluid models, becomes a nontrivial task. There is uncertainty in the literature associated with modeling shear stresses in the liquid phase. In some two-fluid models, in particular, those described in [7, 8], the shear viscosity in the liquid phase is taken to be equal to the viscosity of the pure liquid, while in other models the viscosity is considered to be equal to the effective viscosity of the suspension [5]. Despite the fact that both of these approaches can give close results, strictly speaking, they are erroneous. The single-fluid description of suspensions based on equations for the characteristics of a two-phase flow appears to be a good alternative to two-fluid models. Note that a similar approach using equations for a two-phase medium in general is also used in modeling two-phase flows with low-inertia particles [13].

In the field of hydrotransport theory, great achievements belong to scientists - M.A. Velikanov, who substantiated the gravitational theory of solid phase motion of a suspension-carrying flow, and V.M. Makkaveev with his diffusion theory of pulsating motion of solid particles in a liquid flow. In expanding and deepening the theoretical approach to the problem of solid transfer in a liquid flow, important studies were carried out by eminent scientists such as N.A. Silin, A.P. Yufin, M.A. Dementyev, V.N. Pokrovskaya, A.E. Smoldyrev, A.G. Dzhvarsheishvili and others. Of the foreign schools, a great contribution to the general theory and practice of hydrotransport was made by the works of R. Durand, R. Wooster, D.F. Richardson, S.A. Shchuk, V. Pazhonka, C. Kemblovsky, E. Sobota, P. Slater and others [11-13].

Problems of turbulent flow of liquid with solid particles require consideration of the influence of the solid phase on the turbulent characteristics of the flow. With increasing volume concentration and mass loading of the flow, the contribution of interparticle interactions to the transfer of momentum and energy of the dispersed phase increases, the influence of particles on both average and pulsation characteristics increases. In a two-phase flow, additional effects of production and transfer of turbulent energy arise[14].

The aim of this work is to develop a statistical model of turbulence and a method for calculating turbulent flows of liquid with an arbitrary concentration of solid particles based on a rheological model of a suspension with a quasi-equilibrium velocity of interphase slip. An additional important aspect of the presented model is to take into account the effect of particles on the turbulence of the carrier liquid. Using the developed model, this study calculates pressure turbulent flows of liquid with heavy particles in horizontal channels. Comparison of numerical results with experimental data and with calculations by other authors performed using eddy-resolving methods demonstrates the ability of the model to adequately describe various modes of two-phase flows.

2. Formulation of the problem

The approach to describing the dispersed phase is associated with the introduction of a set of continua, each of which relates to a specific phase of the mixture. Quantities related to the solid dispersed and carrier liquid phases are designated by subscripts and respectively. Non Brownian particles are assumed to

be spheres with constant diameter and constant density. The volume concentration of particles characterizes the proportion of the volume occupied by the dispersed phase.

SST turbulence model.

Menter's shear stress transfer (SST) model [5-6] is a combination of the k-ε and k-ω models. For the wall layer, k-ω is used, for the outer region - k-ε. This model is currently very popular and is included in many CFD packages.

$$\begin{cases}(\mathbf{U}\cdot\nabla)k = \nabla[(\nu+\sigma_k\nu_t)\nabla k]+P-\beta^*\omega k,\\ (\mathbf{U}\cdot\nabla)\omega = \nabla[(\nu+\sigma_\omega\nu_t)\nabla\omega]+\frac{\gamma}{\nu_t}P-\beta\omega^2+2(1-F_1)\frac{\sigma_{\omega2}}{\omega}\nabla\omega\nabla k.\end{cases} \quad (1)$$

Here k is the specific turbulent kinetic energy (m^2 s^{-2}), ω is the specific rate of turbulent dissipation (s^{-1}). Other values are presented in works [5-6].

The following boundary conditions were set for the SST model

$$\omega = 10\frac{U}{L}, \quad k = 0.1\frac{\nu U}{L} \quad (2)$$

Averaging of equations describing turbulent flow of suspension is carried out using the Favre approach or phase averaging. Averaged flow parameters are determined based on the condition of preserving the form of the continuity equation $\langle A\rangle_p = \langle\phi A\rangle / \langle\phi\rangle$ The average values characterizing the mixture are determined by the corresponding sums: density $\rho = \rho_f(1-\phi)+\rho_f\phi$, velocity where and - Favre-averaged velocities of the liquid and particle continuum.

3. Methods

The mathematical model of turbulent flow of a suspension includes the equations of continuity, momentum, and transport of turbulent quantities formulated for the mixture as a whole, as well as the transport equation for the particle concentration and the algebraic equation for the relative interphase velocity. The Favre-averaged equations of mass balance and momentum of the mixture have the form

$$\frac{\partial\rho}{\partial t}+\nabla(\rho U)=0$$
$$\rho\frac{dU}{dt} = -\nabla P+\rho g+\nabla\left(T_v+T_t+T_d\right) \quad (3)$$

Where $\frac{d}{dt}=\frac{\partial}{\partial t}+(U\nabla)$ - substantial derivative, P-average pressure, T_t - Reynolds stress tensor of the mixture, T_v - viscous stress tensor of the mixture, T_d - the so-called diffusion stress tensor. In the equations of mass and momentum balance of a two-phase medium, the terms describing interphase interaction are absent. The Favre-averaged equation of particle concentration transfer has exactly the same form as for instantaneous concentration:

$$\frac{\partial\rho_p\phi}{\partial t}+\nabla\left(\rho_p U\phi\right)=0 \quad (4)$$

The equation for the instantaneous velocity of a continuum of particles is [14]

$$\rho_p\phi\frac{d_p u_p}{dt} = f_D+f_p+\phi\Delta\rho g+\rho_f\phi\frac{d_f u_f}{dt}+\nabla\sigma_p \quad (5)$$

Where, $\Delta\rho=\rho_p-\rho_f$ density difference; $\frac{d_i}{dt}=\frac{\partial}{\partial t}+(u_i\nabla)$ substantial derivative for i -th; f_D -phase interphase force of hydrodynamic resistance; f_p - additional force factors arising from the relative motion of a particle and a liquid; σ_p - stresses in the solid phase.

After averaging the equation for the particle field velocity (5), using the assumptions of small acceleration of the relative velocity $d(u_p-u_f)/dt$ □ 1 and the proximity of turbulent stresses of particles and liquid, we obtain the following expression:

$$\Delta\rho\phi\left(\frac{dU}{dt}-\nabla T_t\right) = \Delta\rho\phi g+F_D+F_L+\nabla\sum p \quad (6)$$

Here, in the right-hand side, the averaged drag force, lift force, and stress of the particle continuum are written in brackets. The averaged drag force is proportional to the average relative velocity:

$$F_D = -\frac{\rho_p\phi}{\tau_p}\langle U_r\rangle = -\frac{\rho_p\phi}{\tau_p}(U_r-u_d), \quad u_d=\frac{\langle\phi' u_f'\rangle}{\phi(1-\phi)}, \quad (7)$$

Where τ_p - the dynamic relaxation time of a particle in a constrained flow; $U_r = U_p - U_f$ -relative velocity; u_d - diffusion velocity arising from the difference between the average velocity of the fluid and the average velocity of the fluid along the particle path (the so-called velocity of the fluid as seen by the particle).

The angle brackets denote the averaging operation, and the prime denotes the pulsating component of the field quantity. As a result, the equation for the average particle velocity is simplified to an algebraic equation for the relative velocity:

$$U_r = \frac{\tau_p}{\rho_p}\left[\Delta\rho\left(g - \frac{dU}{dt} + \nabla T_t\right) + \frac{F_L + \nabla\sum p}{\phi}\right] + u_d \tag{8}$$

The algebraic equation for the relative velocity (8) includes turbulent diffusion, particle stresses caused by interparticle interactions, and the Saffman lift force in shear flow.

The turbulence model was chosen based on the following considerations. The overwhelming majority of turbulence models for dispersed flows are based on a linear model of eddy viscosity, closed by equations for the kinetic turbulent energy and its dissipation rate (k-ε models) or the specific dissipation rate (k-ω models). In this case, as a rule, separate equations are also written for the turbulent characteristics of the liquid and the particle field. The turbulence model was selected based on the following considerations. The overwhelming majority of turbulence models for dispersed flows are based on the linear eddy viscosity model closed by equations for the kinetic turbulent energy and its dissipation rate (k-ε models) or the specific dissipation rate (k-ω models). In this case, as a rule, separate equations are also written for the turbulent characteristics of the fluid and the particle field. The main disadvantage of the linear eddy viscosity model is the inability to describe the anisotropy of turbulent stresses and, as a consequence, secondary flows in channels. A nonlinear two-parameter algebraic model of Reynolds stresses for a dispersed turbulent flow with low-inertia particles was presented in [15]. However, such a model is limited to flows with a low particle concentration.

Interfacial forces

The interfacial resistance force is described by the relaxation time of particles in a constrained flow as

$$\tau_p = \frac{\tau_{p0} f^h}{1-\phi}, \quad \tau_{p0} = \frac{4\rho_p d_p}{3\rho_f C_D \left|\langle U_r \rangle\right|}, \tag{9}$$

Where τ_{p0} -- the dynamic relaxation time of a single spherical particle in an infinite flow; f^h - the function of constrained sedimentation, taking into account the concentration constraint on the resistance force of particles; C_D - resistance coefficient.

For particles moving in a Newtonian fluid, the drag coefficient is described by the Schiller-Neumann approximation [12]:

$$C_D\left(\mathrm{Re}_p\right) = \left(24/\mathrm{Re}_p\right)\left(1 + 0.15\,\mathrm{Re}_p^{0687}\right) \tag{10}$$

where the relative Reynolds number for the dispersed phase is determined by the properties of the carrier phase and the relative velocity - $\mathrm{Re}_p = \rho_f d_p \left|\langle U_r \rangle\right| / \mu_f$

The results of experimental studies devoted to the study of the influence of turbulence of the carrier medium on the interphase force remain contradictory. In a number of experiments, an increase in the average resistance force is reported, while in others, on the contrary, its decrease is reported [19]. An approximate version, taking into account the contribution of the pulsating sliding of particles relative to the liquid to the resistance force, consists of calculating the relative Reynolds number using the formula:

$$\mathrm{Re}_p = \rho_f d_p \left|\langle U_r \rangle + \left(1 - f_p\right) k_f\right| / \mu_f \tag{11}$$

In the equation for the relative velocity, we can isolate the diffusion part and rewrite the equation for the conservation of mass as a convective-diffusion equation for the transfer of the volume concentration of particles:

$$\frac{\partial \rho_p \phi}{\partial t} + \nabla\left[\rho_p \phi\left(U + \frac{\rho_f}{\rho}(1-\phi)U_r + \frac{1}{\phi} D_\phi \nabla\phi\right)\right] - \nabla\rho_p D_\phi \nabla\phi = 0,$$
$$D_\phi = \frac{\rho_f}{\rho}\left((1-\phi)\frac{\tau_p}{\rho_p}\frac{\partial \Pi_p}{\partial \phi} + \frac{\nu_t}{\sigma_\phi}\right) \tag{12}$$

Where D_ϕ -effective diffusion coefficient, including turbulent diffusion and the diffusion contribution of interparticle interactions.

The parabolic equation requires setting boundary conditions for concentration at all boundaries. On solid walls, conditions of no flow and no adhesion are set for the velocity and the absence of particle flow through the wall - $\frac{\partial \phi}{\partial n} = 0$. The effect of partial sliding of the suspension on the walls is not taken into account in this work.

At the outlet of the network section with a heat exchanger.

The total second heat loss in the section with the heat exchanger was determined by the formula

$$Q_{tepl} = \rho c_B Q_A \left(T_н - T_к\right)[W] \tag{13}$$

4. Results

A simplified version of the model without the contribution of particles to turbulent motion was used for numerical simulation of steady-state turbulent flow of a hydraulic mixture in a horizontal circular pipe for several cases for which experimental data exist [7–9]. Flows with a high flow loading degree were considered: from 10 to 40% by volume content of the dispersed phase at high velocities. Particle diameters varied from 90 to 520 μm, and pipe diameters were 50–100 mm. Comparison with experimental data demonstrated the ability of the model to predict well the distributions of particle concentration and suspension velocity along the channel height for a heterogeneous flow regime. However, the model overestimated the longitudinal pressure drop for flows with a high flow loading. Good agreement with experiment was observed only for flows with an integral particle concentration of < 20%.

In this paper, the first test is a fully developed flow of water with sand in a horizontal circular pipe, studied both experimentally [8] and by numerical simulation [10]. The channel diameter is D = 51.5 mm, the average diameter of sand particles is dp = 165 μm, and the particle density is ϱp = 2650 kg/m3. The settling velocity of a single particle is us = 0.0185 m/s, and the Reynolds number is Rep(us) = 3.06. The experiment investigated a flow with an average velocity of Ub = 1.6 m/s and an average volume concentration of C = 8.4%.

Figure 1(a-b) portrays the solid velocity profile in xy plane at x = 3.7 m from the pipe inlet at solid concentration, C = 10 – 20% for different mean flow velocity range, V = 3-4 ms^{-1}. It has been found that solid particulates exhibit maximum velocity at the center of the pipeline and least at the pipe wall. The velocity profile is flatten in nature at C = 10% for all mean flow velocity range.

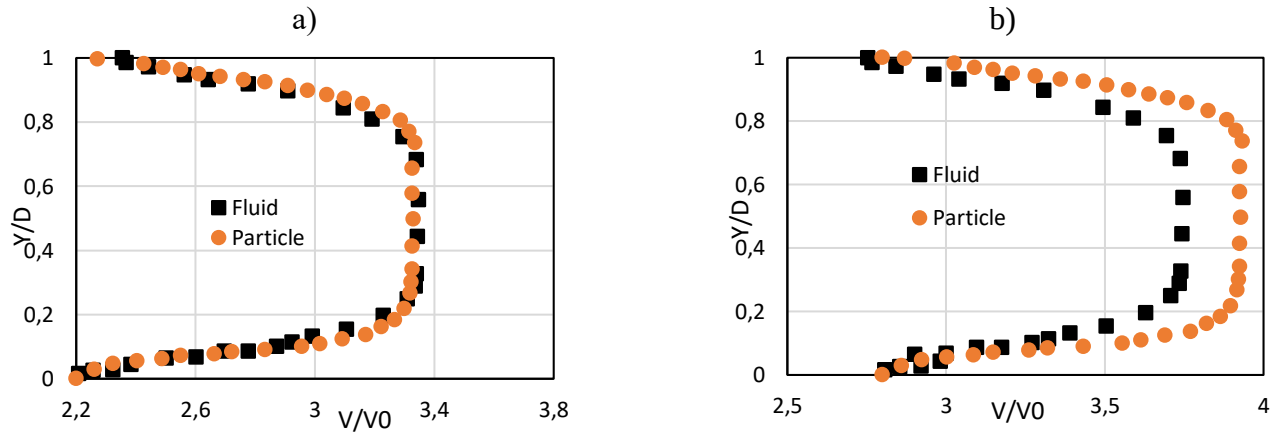


Figure 1. velocity profile longitudinal velocity of the fluid and particle (a) V=3 ms^{-1}, (b) V=4 ms^{-1}.

Figure 2 (a-b) portrays the solid concentration profile in horizontal in xy plane at x = 3.7 m from pipe inlet, at different mean flow velocity range, V = 3-4 ms-1 and solid concentration range, C =10-20%. The findings show that the solid concentration is maximum at the bottom of the pipe and subsequently decreases from the bottom to the top of the pipeline at all mean flow velocity and solid efflux concentration range..

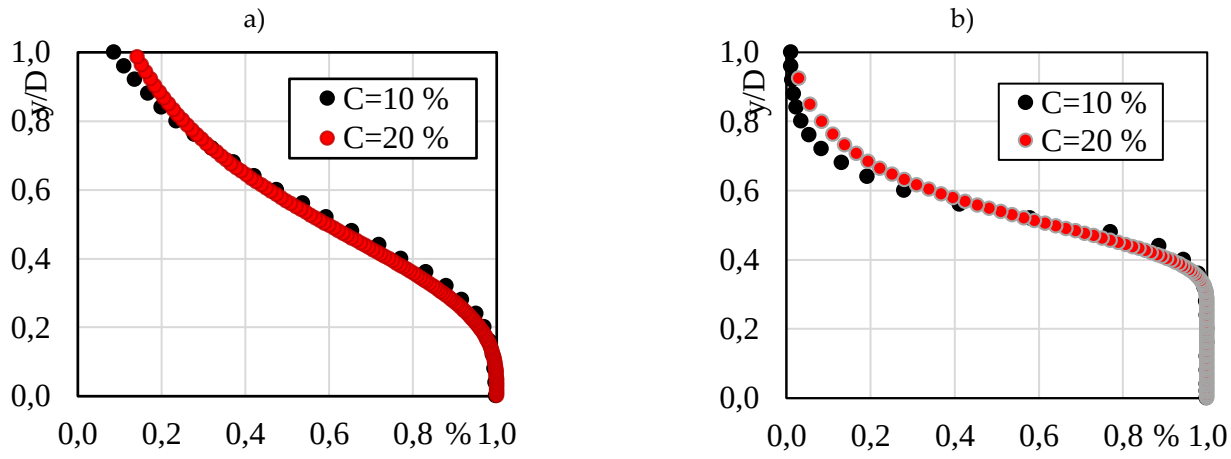


Figure 2.solid concentration profile at (a) V=3 ms-1 , (b) V=4 ms-1

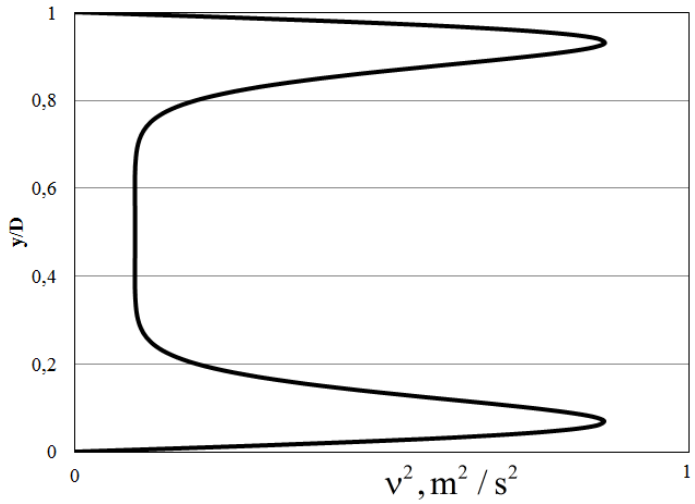


Figure 3. Kinetic energy of turbulent pulsations

5. Discussion

A model for describing the turbulent flow of a highly concentrated mixture of Newtonian fluid with solid particles is presented. The peculiarity of this model in comparison with two-fluid models is that it solves the system of equations for a two-phase flow as a whole using rheological relations and transport equations for the concentration of the dispersed phase with an algebraic equation for the interphase slip velocity. The rheological model describes the viscous and contact modes of particle motion in a liquid.

6. Conclusions

A multiparameter version of the SST turbulence model is used to simulate suspension turbulence, taking into account the near-wall anisotropy of turbulence. The turbulence model is supplemented by the contribution of particles to turbulence generation due to the average slip velocity and dissipation associated with viscous losses when particles are involved in turbulent motion.

The model under consideration has been tested on problems of steady-state turbulent flow of a suspension with heavy particles in a horizontal pipe. RANS modeling demonstrates that without taking into account the modulation of turbulence by particles, the model significantly overestimates turbulent pulsations in the near-wall layer with a high concentration of particles adjacent to the lower wall. Taking into account the dissipation of turbulence due to the involvement of particles in turbulent motion allows us to adequately describe the turbulence intensity and the development of a stationary layer of settled particles. Analysis of secondary flows in the transverse plane allowed us to detect a three-layer structure of a two-phase flow. However, to clarify the flow structure and improve the description of turbulence anisotropy, it is necessary to use second-order closure models. The obtained results demonstrate good predictive ability of the model both in relation to local distributions of average and pulsation quantities, and in relation to the integral dependence of the pressure drop on the mixture velocity.

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